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Welded Continuous Frames and Their Components

Interim Report No. 26

RULES OF PRACTICE IN PLASTIC DESIGN

by

Lynn S. Beedle and Bruce G. Johnston

(not for publication)

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RULES OF PRACTICE IN PLASTIC DESIGN

Sequel to: "An Evaluation of Plastic Analysis as Applied to Structural Design"

Lynn S. Beedle and Bruce G. Johnston

(Not for Publication)

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Rules of Practice in Plastic Design

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SYNOPSIS

The evaluation of a considerable amount of research work has demonstrated the applicability of plastic analysis to structural design. It is therefore desirable to consider tentative specifications and, as a preliminary basis, this report of suggested rules of practice is prepared. Design problems are described, the applicable results of research are discussed, and rules of practice are suggested in as specific terms as are now possible. The necessary additional research is outlined.

1. INTRODUCTION

1.1 STRUCTURAL DESIGN

In the process of selecting suitable members for a steel frame structure it is necessary to make a general analysis of structural strength and, second, to examine certain details (usually covered by codes or specifications) to assure that local failure does not occur but that the structure will meet its intended function.

The structural strength or design load of a steel frame may be determined or controlled by a number of factors, any one of which may actually constitute a "Limit of Structural Usefulness":

1. Attainment of a hypothetical yield-point stress (ignoring stress concentrations)
2. Brittle fracture
3. Fatigue (endurance limit)
4. Instability
5. Attainment of maximum (plastic) strength
6. Large Deflections

Item 1 in conjunction with Items 3 and 4 has, for many years, been the basis for conventional structural design which uses the "working stress" concept. The provisions of specifications are intended to insure that one of the other "limits of structural usefulness" does not embarrass the structure.

1.2 PLASTIC DESIGN

Strictly speaking, a design based on any one of the six factors listed above could be referred to as a "Limit Design", although the term usually has been applied to determination of ultimate strength according to Items 4 and 5.(35) "PLASTIC DESIGN" as an aspect of limit design embraces primarily Item 5 (attainment of maximum plastic strength) as applied to continuous beams and frames. Items 4 and 6 ("Instability" and "Large Deflections") must also be considered. It is, first, a design on the basis of the maximum load the structure will carry as determined from an analysis of strength in the plastic range (i.e., plastic analysis). Secondly, it consists of consideration by rules or formulas -- of certain "limitations", "restrictions", or "modifications" that would otherwise prevent the structure from attaining this theoretical maximum strength. Many of these same limits are present in conventional design (brittle fracture, fatigue). Others are inherently associated with the plastic behavior of the structure. The unique feature of plastic design is that the ultimate load rather than the yield stress is regarded as the design criterion.

What are the assumptions in plastic design? Maximum load computations for continuous frames are based on the assumption that "plastic hinge moments" are developed at critical points in the structure and maintained during the subsequent load history. Thus conventional design criteria for the stability of details, which merely guard against <u>elastic</u> buckling, require reexamination in plastic design where plastic buckling must be minimized. The further assumption, or requirement, that the deflection be tolerable under the conditions of use, indicates a second design criterion.

The maximum load determined by plastic analysis may then be thought of as an "ideal maximum" as there is the possibility that one or more of a number of factors may operate to make it impossible of attainment; still other factors will operate to increase this maximum load. These factors have been reviewed and evaluated in a recent report,[1] and it is evident that certain "plastic parameters" are sufficiently substantiated so that rules of practice may now be laid down in order that engineers may begin to enjoy the economies inherent in the method. Some rules may be tentative, but, particularly in view of the fact that British engineers have made use of the "Collapse Method" of design,[1,8] it is believed that recommended practice may be suggested for use in the design of a wide range of structures.

1.3 OBJECTIVE

It is the purpose of this present paper to make preliminary suggestions leading to rules of design practice that could be adopted to insure that structures will realize their potential maximum load capacities.

The design problems to be discussed are as indicated in the Table of Contents. Tentative rules of practice will be suggested as examples of possible approaches to meet each problem. At present, some of the rules are based on incomplete test results and approximate theories. It is recognized that much research is now underway that undoubtedly will lead to improvements in the future.

It is also considered that this report would be the basis upon which a tentative specification for plastic design might be written. In this respect the paper summarizes methods for plastic analysis and indicates the justification for the assumptions made therein. Where additional work seems desirable, this is indicated.

As a means of implementing the use of plastic design, a paragraph (or paragraphs) could be added to present specifications allowing the designer to proportion certain structures according to the plastic methods of structural analysis so long as he demonstrates that it is in accordance with the recommended rules of practice. An alternate specification can then be developed to which specific reference would be made when available.

2. GENERAL PROVISIONS

Rules of practice for plastic design should include certain general provisions covering types of construction, materials, fabrication, and welding. To some extent these have been discussed in the previous paper evaluating plastic design and will not be considered in detail in this report, although statements regarding them should be formalized for adoption in a code.

2.1 TYPES OF CONSTRUCTION

Plastic analysis obviously should be applied in the design of those structrues and structural components which have been studied by analysis and/or tests sufficiently to give confidence that the calculated maximum strengths will be realized. Undoubtedly the scope of application will broaden in the course of time, and the specifications should permit this on the basis that adequate tests and analysis assure that the structure will support the calculated loads.

At present it would appear that application should be made to all-welded continuous frame construction using wide-flange beams or American Standard sections within limits that will be tentatively suggested in this report. Members built up by welding would be suitable within the same dimensional limitations.

RULE OF PRACTICE

The following types of construction using fully continuous welded construction are considered as appropriate:

1. Industrial building frames (one story, single or multi-span)
2. Tier building frames (two or more stories)
3. Structures intended to absorb dynamic load (bomb burst, collision, earthquake)

FURTHER RESEARCH: As described later, when it is demonstrated that continuous joints can be economically proportioned using bolts, then plastic design could also be applied to such structures.

First application to tier buildings may well lie in the plastic design of the beams only, except in the upper one or two stories where the entire framework could be proportioned by the plastic methods. Studies are required.

Application to single span frames, flat roofed, has been confirmed by test. Tests that will follow are for gabled roofs.

2.2 MATERIALS

The material would be structural grade steel for buildings and bridges, ASTM A7, with consideration given to any modifications in specifications that would tend to insure weldability and ductility of behavior after welding. Modifications in the structural steel specifications that have been suggested by the American Welding Society(9) should be followed.

RULE OF PRACTICE
ASTM A7 steel for bridges and buildings (or as modified to insure ductility and weldability) shall be used.

2.3 FABRICATION

With respect to fabrication of material to be welded, stress concentrations should be avoided and the fabrication process should be such as to promote ductility. Consideration should be given to the question as to whether or not sheared edges should be permitted. The adverse effect of sheared edges on ductility beyond the yield point is currently being investigated at the University of Illinois.

RULE OF PRACTICE
Fabrication process should be such as to promote ductility. Sheared edges and punched holes in flanges are not permitted.

2.4 WELDING

Insofar as is necessary, welding processes should be specified so as to eliminate the possibility of fracture in the weld metal itself. (Art. 5.1)

RULE OF PRACTICE
The provisions of the Specifications of the American Welding Society should be followed.

2.5 METHODS OF ANALYSIS

There are three methods by which the maximum plastic strength of a continuous structure may be directly determined without the need for the necessarily involved elastic or elastic-plastic analysis:

(1) EQUILIBRIUM method (semi-graphical)[8]

(2) Method of INEQUALITIES[10]

(3) Method of VIRTUAL DISPLACEMENT[11]

A specification need not direct that any particular one of these methods be used, but it should call for an analysis giving an accurate measure of maximum strength.

It is considered that Method 3 (VIRTUAL DISPLACEMENT) is simpler for the more complicated structures. The EQUILIBRIUM Method (No. 1) works very well for a simple structure (i.e., continuous beam or single span frame) and gives an excellent physical picture to parallel the analysis. Unfortunately, it becomes very involved for more complex structures. The method of INEQUALITIES is considered too involved for present application to design practice.

The VIRTUAL DISPLACEMENT Method is therefore recommended as being generally applicable, being subject, of course, to the other provisions outlined in this paper. In Appendix 1 the Equilibrium and Virtual Displacement methods of analysis are described more fully.

Plastic methods of analysis are based upon the formation of "plastic hinges", and it is assumed that arbitrary angle changes are possible at the critical sections while the plastic moment is maintained. In the first part of Appendix 1 the reserve of strength due to this so-called "hinge action" is illustrated.

> RULE OF PRACTICE
>
> The design shall be based upon the maximum strength as determined by a suitable method of plastic analysis. The equilibrium method is satisfactory for continuous beams. For more complicated structures the virtual displacement method is appropriate.

FURTHER RESEARCH may simplify plastic analysis techniques.(15)

2.6 DESIGN PROCEDURE

> RULE OF PRACTICE
>
> The design procedure is to be as follows:
>
> 1. The working loads are determined, both for dead and live load and for dead load plus live load plus wind.
> 2. The "Full Plastic Load"* (the load at maximum plastic strength) is determined by multiplying the working loads by the load factors of safety, F, (see Art. 3.4).
> 3. An estimate is made of the ratio of M_p values for the various members of the frame.**
> 4. By one of the methods of analysis, M_p is determined together with the moment diagram.
> 5. The shapes are selected from the Handbook.
> 6. The design is checked to see that it complies with the "Rules" to follow.

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* Full load = Working load x Load factor of Safety
Full plastic load = predicted maximum plastic strength

** Heyman (12) has treated the problem of design for minimum weight.

3. LOADS AND FORCES

3.1 GENERAL DISCUSSION

In plastic design the relative importance and treatment of dead-load, live load, and impact will be somewhat different from conventional design. The most certain application of plastic design would be for those structures in which the loads are primarily or wholly immovable, or equivalent to "dead load". In the consideration of live load, the number of repetitions of load as well as their maximum magnitude may be of greater importance than in the case of elastic design.

This suggests two general types of loading:

(a) Proportional Loading
(All loads increase in a constant ratio, one to the other, and without repetition, to a maximum value)

(b) Variable Repeated Loading
(Individual loads may vary independently, each within its own prescribed maximum and minimum limits)

An important area that is a special case of variable loading is for structures designed for peak snow load or peak wind load (the improbability of both occurring simultaneously would be of considerable importance). The loading would not be "proportional", but repetitions of peak load would be so low in number as not to require consideration of its cyclic nature.

3.2 PROPORTIONAL LOADING

Proportional loading is the type usually considered, and formed the basis for the analysis in Appendix 1. This designation covers two aspects of the loading:

(1) If the different loads are denoted by

$$P_1, P_2, P_3 \dots\dots\dots P_i$$

then the ratio of the loads are constant, or

$$\frac{P_i}{P_{i+1}} = \text{Constant}$$

(2) The loads increase steadily to a maximum value, that maximum value being realized a relatively small number of times in the life of the structure.

> RULE OF PRACTICE
>
> The analysis shall be based on an assumption of proportional loading.

FURTHER RESEARCH is needed to indicate the limits of number of cycles and magnitude of load for which this rule must be modified. (see Art. 3.3)

3.3 VARIABLE REPEATED LOADING (Non-Proportional)

The maximum load computed by the plastic methods of structural analysis is independent of the method by which the individual loads are brought up to their final values only if failure under some other load system does not previously occur. There are other load systems for which such "failure" may occur and this is the subject of Chapter 4.

3.4 LOAD FACTOR OF SAFETY

The concept of the "load factor of safety" as compared with the "stress factor of safety" or "limiting working stress", has been effectively described by Pugsley.[16] In essence, as long as the structure will carry, throughout its expected life, loads that may normally be assumed to act, then it does not matter what the stresses may be.[1]

Such a concept is at the heart of plastic design. The questions remain as to the magnitude of the load factor and the magnitudes of the loads to be assumed in the design. Inevitably the future will see more attention paid to a statistical evaluation of the magnitude of load, to its rate of application, and to the design of structures with finite limit on their useful life.[17]

Considerable further research is essential and is going on at the present time in the Lehigh project. From the long range point of view it has been suggested[16] that the following studies must be carried forward continuously by the engineering profession:

(1) Accident rates on existing structures
(2) Full-size experiments on structures
(3) Surveys of external loads.

A more immediate problem is that of selecting the magnitude of the load factor of safety. It is conceived as being made up of two parts:

(1) possibility of increases in loading above those assumed (and these will be different for dead and live load and for the different types of live loading),

(2) certain "variables", inherent in the material, affected by construction and neglected in analysis (see Ref. 1 and Ref. 2, p. 56) characteristics of which are:

(a) variation in dimensions

(b) variation in physical properties

(c) approximation in stress analysis

(d) errors in fabrication and erection (workmanship)

(e) loss of section due to corrosion

(f) time effects.

Modern methods of handling such a problem will be to determine the uncertainty of each type of loading and of each "variable" (Item 2 above). The different percentage variations may then be weighted and combined by probability to give a load factor of safety for dead load and for the different types of live load.

The maximum load, P, for which the structure must be designed is then determined from,

$$P = F_d P_d + F_{L1} P_{L1} + F_{L2} P_{L2}$$

where

F = Load factor of safety

Subscripts = designations for dead load (d) and live load (L) and types of live load (L1, L2)

As is done in the case of impact, it is already common practice to apply "factors" to different loads in order to account for some form of structural behavior that does not lend itself to ready analysis.

Having made this statement about the load factor of safety, it is regrettable that no positive rule of practic may be offered that results from an analysis based on the methods just outlined. The provisional rule must therefore be based on present practice and the simple beam forms a fair basis for comparison.

The conventional design of a simple beam by AISC Specifications carries a factor of safety of 1.65 against "failure" as defined by the attainment of yield-point stress,

$$P_W = P_y/1.65$$

If the two working loads are to be made equal (i.e., if the reserve of strength above working load in a rigid frame is to be the same as now provided in a simple beam), then, since on the average the shape factor, f, of WF shapes is 1.14, the working load would be given by the expression,

$$P_W = \frac{P_y \ (f)}{F}$$

Substituting for P_W the value $P_y/1.65$, the load factor of safety, F, would be

$$F = \frac{P_y \ (f) \ (1.65)}{P_y} = 1.65 \times 1.14 = 1.88$$

The above analysis presumes that the conventional stress factor of safety was selected with full knowledge and consideration of the reserve in bending of a beam beyond the elastic limit. This is believed not to be the case. Tension members are also designed at 20 ksi working stress (F = 1.65), and bending members under test may actually exhibit a reserve of strength of less than 10% beyond the yield point unless properly supported. A fair factor of safety would in this case be 1.65.

A median value of F = 1.75 applied to dead load and live load is suggested provisionally, being consistent with the recommendations of British investigators.

For live load + dead load + wind, the present specifications allow a 33 1/3% increase in stress. A similar allowance for a load factor of safety would give

$$F = 1.75 \times (3/4) = 1.31$$

A value of the load factor of safety of 1.40 is suggested provisionally for this case.

RULE OF PRACTICE (Tentative)
For building construction a load factor of safety of 1.75 is to be applied against dead load and live load.
For DL + LL + Wind, F = 1.40.
Loadings are to be those presently specified by the American Institute of Steel Construction.

FURTHER RESEARCH is needed as outlined earlier in this article. A complete study of the problem would be most helpful.

3.5 YIELD STRESS LEVEL

Ref. 1 contained discussion of the yield stress (starting on page 226-s). The premise is that:

(a) The mean or average stress will be taken as the basis for structural design, with the factor of safety to take care of variations. This average is determined from a consideration of the full cross-section.

(b) The yield stress level (lower yield point) determined at "static" rate is the significant property as against the upper yield point.

Certain data are available(1) and current studies(18) are providing more precise information. They suggest that the proper value will be obtained from the mean value of the ASTM acceptance test data to which is applied a factor of about 15%. (It is a coincidence that this gives an answer that is the same as the present minimum yield point value.)

RULE OF PRACTICE (Tentative)
The yield stress level will be taken as 33,000 psi in bending, compression, and tension and will be the basis for computing the full plastic moment.

FURTHER RESEARCH may allow an increase in this value to as much as 37,000 psi for ASTM A7 steel on the basis of a correlation between ASTM acceptance tests and the strength of full cross-sections of steel members.

3.6 IMPACT, BLAST, AND EARTHQUAKE

In conventional design impact is taken care of by a load increase factor that is intended to insure that the stresses will stay in the elastic working range. In the case of plastic design, if the philosophy of basing design on maximum strength is carried through consistently, special dynamic analyses would have to be made for each type of design application to include the effect of time-rate of increase of load and its duration. In other words, the load-time function would be evaluated as well as the plastic resistance function of the structure. This sort of analysis is commonly made in special studies of bomb or earthquake-loaded structures. An estimate of increased yield stress should be applied in analyses under impact load and these increases should be consistent with the rate of strain predicted during the plastic range in the structure under consideration. For example, an increase of 30 percent in a basic estimated yield stress level of 33,000 psi would be appropriate for a structure under blast load

RULE OF PRACTICE UNDER CONSIDERATION

4. VARIABLE LOADING

4.1 TYPES OF VARIABLE LOADING

Analysis for "proportional loading" (which is the basis of routine plastic design) assumes that the structure is loaded statically or with a simultaneous application of the various loads. There are many structures in this category and therefore the immediate field of application for plastic design is by no means a restricted one.

As was mentioned in Chapter 3, it is theoretically possible for a structure to "fail" due to variable loading (the general term chosen to include "cyclic loading", "repetitive loading" or other forms of non-proportional loading) at a load which is less than the full plastic load. However, it would be wrong to say that just because one or more of the loads was variable, the structure could not be designed by the plastic methods. Undoubtedly there are certain limits within which the structure may be assumed to be loaded proportionally, even though some of the forces are cyclic. It is of interest that the British plastic design procedure dispenses with the problem by:(8)

> "Where rolling or alternating loads occur, special treatment is required. This document deals with static forms of loading only."

The following material has been prepared in the belief that plastic methods of structural analysis will be applicable to many additional design problems that would otherwise be placed improperly under the broad heading of "Fatigue". Discussion in Ref. 1 starts on p. 233-a.

The general types of loading sequence are (1) Proportional Loading (Art. 3.2), (2) Fatigue, (3) Plastic Fatigue (or Alternating Plasticity), including reversal of stress of limited number of cycles, and (4) Variable Repeated Loading (leading to the problem of "Deflection Stability", "Progressive Collapse", or "Shakedown").

Questions that require evaluation in a design for variable loading are:

(1) The number of cylces of each of the loads.

(2) The range of variation of load (maximum and minimum values).

(3) The variation of stress in the particular structural member -- and in other frame components -- as a result of the cyclic nature of the loading and in combination with other (dead) loads.

This latter aspect is particularly important because the mere fact that some loads reverse (or repeat) does not automatically require that the entire structure be designed according to fatigue limits. For the most efficient design, the effect of repeated loading on the whole structure, together with other loads must be considered.

RULE OF PRACTICE

Where certain of the loadings are variable (cyclic) in nature, the number of cycles, magnitude of maximum and minimum values, and the overall influence (in combination with dead loads) on structural members will be determined, and design will be in accordance with the rules in the following sections.

4.2 FATIGUE

Failure by fatigue is generally associated with elastic behavior. It is characterized by one or possibly more loads of a system being subjected to a very large number of cyclic variations.

According to Lazard[19] cyclic fatigue loading may be classified under the following headings:

(a) Fluctuating +A to +B
(b) Repeating 0 to +B
(c) Alternating -A to +B
(d) Oscillating -B to +B

These are illustrated in Figure 1. The term "Reversed loading" may be more descriptive for Item c.

Very recent interpretive reports on the matter of fatigue are contained in Refs. 20 and 21.

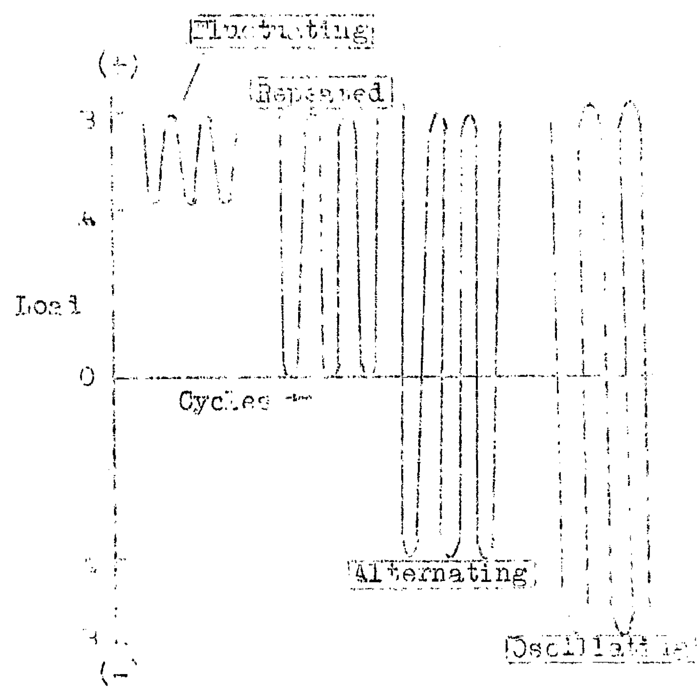


Fig.1

When the loads vary as shown in Figure 1, some members will experience a corresponding change in "stress" or bending moment while others will experience variations much less severe. It is most important to consider this difference because the allowable load in a member for a certain number of cycles will be consistently higher in the case of "fluctuating" moments than for the same number of cycles of "oscillating" moments. Wasteful design is the result of treating all variable loading as if such loading introduced oscillating moments into a member.

Design for true fatigue is a matter of elastic analysis and not a matter of plastic analysis. What is required is a specification of those loading conditions on the building or other structure that will result in fatigue failure. Answers are not yet completely available; the solution involves the following:

(a) What variation in moment and direct stress will structural members (in particular the welded connections) support for an unlimited number of cycles

(b) What are the limiting combinations of live load and dead load below which fatigue failure cannot occur -- and hence need not be considered.

> RULE OF PRACTICE (Tentative)
>
> Where variations of stresses are such that fatigue failure will occur (limits not yet generally specified, but probably to be expressed as a function of the ratio of variable live load to dead load) then the existing provisions of specifications will be followed.

The above rule of practice does not represent a serious limitation to plastic design because it is considered that ordinary building structures will not normally be subjected to fatigue phenomena. Rather the question is when must a structure be designed according to fatigue?

FURTHER RESEARCH has been outlined, generally, above.

4.3 PLASTIC FATIGUE (Alternating Plasticity)

A far more germane question to plastic design is the following: "How many cycles of loading will the structure sustain at the predicted ultimate load?" This is one of two further matters that must be considered, the other being "deflection or stability" (Art. 4.5).

Only a few test programs have been completed on this topic. However, the results suggest that the allowable number of cycles at the full load is relatively large. Tests at Lehigh[28] were made on small specimens of A201 steels under auspices of the Pressure Vessel Research Committee of Welding Research Council. (The material had a yield point of 36,000 psi and the composition was C = 0.14%, Mn = 0.36%, Si = 0.20%.) The program included variables of surface preparation and heat treatment. The tests indicate that up to the yield point for full reversal (oscillation), 100,000 cycles may be carried in the absence of severe stress-raisers. When the strain is increased to as high as five times the elastic limit strain (corresponding to curvature of at least 5 times the elastic limit curvature), the number of cycles was still above 2000. This is encouraging since few members are subject to full reversal at maximum load intensity.

Our next work in this respect should consist of tests on full component members (girders and connections); this can be done with equipment now being procured for the new Fritz Laboratory. The influence of dead load is precisely the same as described in Art. 4.2 (FATIGUE) and consequently certain plastic fatigue tests on frames are justified.

As has been discussed in Art. 4.5 below, an aspect of Plastic Fatigue is one of two types of successive deformation and has been called "Alternating Plasticity". Typical behavior is illustrated by Figure 2. At a given cross-section there is a variation of moment under cyclic load such that deformation occurs on each cycle. In general the maximum ϕ (and, thus, deflection) would quickly be reached forming the hysteresis loop ABCD. For such loading the question at issue is somewhat different from that raised in the first paragraph of this article. What are the load limits below which the behavior will be elastic (as shown by EF)?

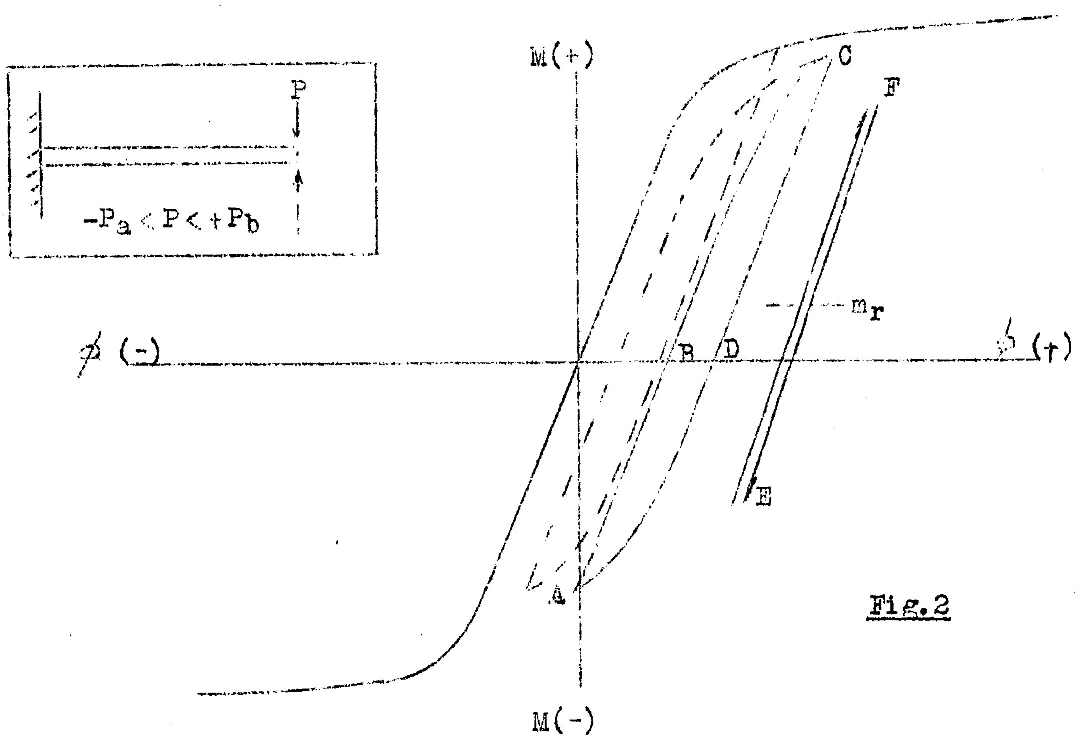


Fig.2

The number of tests are limited, but it appears that the deformation will quickly stabilize as a hysteresis loop (ABCD) when the load is somewhat greater than the theoretical critical load. Fracture will eventually occur after a sufficient number of cycles. If the number of cycles expected in the life of the structure is greater than this number, then it would be necessary to require completely elastic behavior as indicated by E-F in Figure 2. An elastic analysis would be required.[13] The method for considering this in practice might be based, again, on the ratios of live load to dead load.

RULE OF PRACTICE (Tentative)
The full design load will be increased by an appropriate factor whenever the number of cycles of variation in critical moments would otherwise cause plastic fatigue at the full plastic load. (Limits and percentage increases not yet specified.)

4.4 MOVING LOADS

These "Rules" have not been developed as yet to apply to structures with moving (rolling) loads although the general procedures will be similar to those described in this Chapter.

4.5 STABILITY OF DEFLECTION

The previous articles have dealt with the possibility of variation in moment at the same cross-section due to cyclie loading. The design limit has been fracture at that cross-section.

An entirely different mode of "failure" may occur due to variable repeated loading. It is characterized by loss of deflection stability in the sense that the deflections at a particular point increase without limit as the loading system goes through its possible cyclic variations.

The term "Shakedown" has been used by investigators at Brown University and the University of Cambridge(13) to describe the limiting load above which the deflections will not stabilize at a constant value. The term "Deflection Stability" has certain advantages over the term "Shakedown". Recent publications have appeared(19,23) and the early work on the subject was done by Bleich and Melan.

If each of the loads may vary in magnitude independently of the others and each within its own prescribed maximum and minimum limits, then progressive or cyclic plastic flow may result. The question is, does the progressive deflection stop after a few cycles or does it continue indefinitely? If it continues, the structure is "unstable" from a deflection point of view, even though it sustains each application of load.

It follows that the maximum allowable or useful load is one such that, after possibly a few initial cycles of loading, the behavior is elastic. Above this load, one of two types of deformation may occur. In the one case, there will be alternating plastic action at a given section as the loads go through their cyclic variations ("Plastic Fatigue" or "Alternating Plasticity"). In the other, the deflections will continue to increase in the same direction step-by-step and without limit; i.e., there is no stability of deflection. The term, "Incremental Deformation" has been applied to this second mode of successive plastic deformation.

The limiting load above which there is either plastic fatigue or incremental deformation has been called the "shakedown" load by other investigators.(13) Although the two types of deformation are both caused by the fact that the loading is variable, the behavior is different, as is the analysis to predict the critical load. It is for this reason that "Alternating Plasticity" was treated in Art. 4.3, and "Incremental Deformation" is treated under the heading of Stability of Deflection.

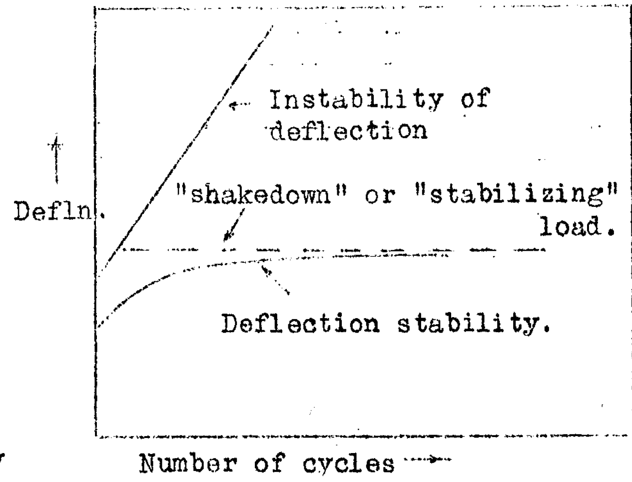


Fig.3

Loss of deflection stability by "Incremental Deformation" is characterized by the behavior shown in Figure 3. The "Stabilizing Load" is also shown. An illustrative example is sketched in Figure 4. When only load P_1 is applied (Loading "A"), yielding will first occur in the positive sense at Section 1. On the other hand when load P_2 is also added (Loading "B") then yielding will occur in the negative sense at Section 2. Then, when load P_2 is removed yielding will again occur in the positive sense at Section 1, and so on. Stability of deflection is lost, because at each suc-

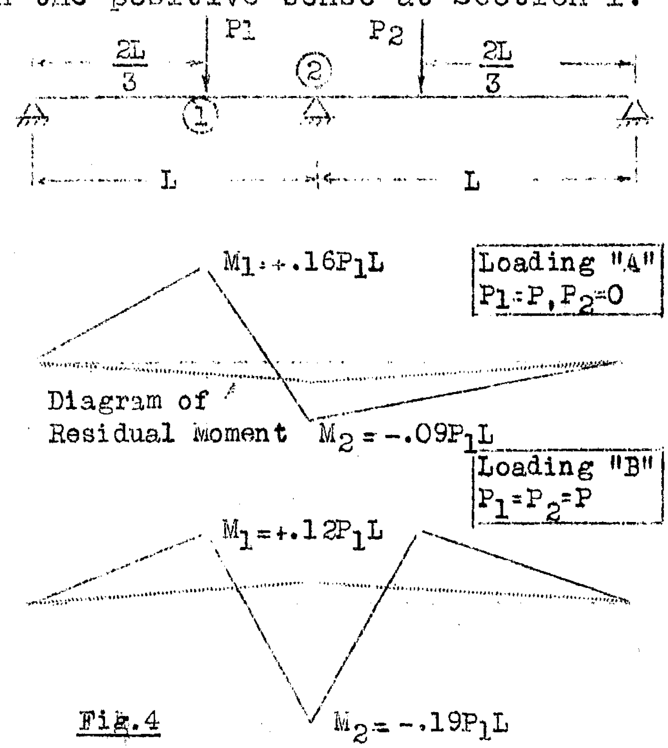


Fig.4

cessive application of loadings "A" and "B" the deflection at Section 1 steadily increases.

Methods of analysis to predict the critical load are not complex, but they require an elastic analysis of the structure which it is desirable to avoid. Symonds and Neal[13] suggest that the ratio of the full plastic load, P_p, to the stabilizing load, P_s, is generally less than 1.25 and that the different load factors of safety will be such that no "modification" will generally be required. However, they suggest (and we concur) that much additional work needs to be done. Tests are now underway on the Lehigh program.

> RULE OF PRACTICE (Tentative)
>
> The full plastic load is to be modified according to analysis of deflection stability ("shakedown") whenever the loading sequences are such as to cause either plastic fatigue (alternating plasticity) as covered in Art. 4.3 or incremental deformation. The limits of ratio of variable live load to dead load for various proportions of structural members are yet to be specified.

5. WELDS

5.1 WELD TYPES

The types of welds that are permitted in plastic design should be carefully defined so as to include only those that will develop full yield strength of the adjacent unwelded material.

Stresses in welds should be calculated so as to be below a value that might lead to brittle fracture of the weld material.

This means that intermittent welding must not be permitted in a region in which plastic hinges might occur. Fillet welding, where continuous and in accordance with AWS practice has demonstrated most satisfactory performance in static tests into the plastic range. Full penetration butt welds have demonstrated adequate behavior.

RULE OF PRACTICE
The applicable procedures of the American Welding Society Code will be followed. Continuous welds are mandatory in all critical joints.

5.2 DETAILS

The design of welded joints must be such as to eliminate severe stress-concentration. An example of an undesirable detail was in the "sniped" stiffeners used on one of the corner connections tested early in the program (Conn. P.)[26] These produced a crack that would not have occurred had the coped plate been welded up.

If flange cover-plates are added to increase the bending stiffness and strength of a beam, the ends of the cover-plates should be detailed so as to provide a gradual transition and, in general, all sudden changes in cross-section should be avoided so as to minimize the possibility of brittle fracture under shock load.

Since provisions must be made for considerable plastic deformation in the connected members, the joints must be designed in such a way that stress concentrations will not develop later due to this deformation. This requires attention to details but constitutes no undue restriction on plastic design.

RULE OF PRACTICE
Joints should be designed to eliminate the possibility of adverse stress-concentrations.

6. DEFLECTION CONSIDERATIONS

6.1 IMPORTANCE OF DEFLECTIONS

One of the common points of uncertainty in structural design has to do with deflections. Although the primary design requirement is that the structure carry the load, it is important that it not deform too much out of shape. A question just as applicable to elastic and plastic design is, "What is the maximum allowable deflection". Unfortunately there is no logical rule available. As discussed in Ref. 1, p. 232-s, the origin of current specification clauses that limit deflections directly or indirectly is obscure, and it is known for example that the deflection limit of structural usefulness of 1/360th of span length does not result from a rational approach to the problem. "More realistic deflection design criteria are needed to supplement existing specifications and to define those conditions under which it is necessary to consider deflection in the application of plastic design".(1)

The problem is not a serious one to plastic design, because in most cases a structure designed for ultimate loading by the plastic methods will actually deflect no more at working loads (which are nearly always in the so-called elastic range) than a structure designed elastically according to current specifications. A plastically designed continuous beam exhibits <u>less</u> deflection than a simply-supported beam designed to carry the same loads.

The load factor of safety does not preclude the possibility that the full plastic load will be reached. So, if deflections are critical computations may be required. Methods are available by which this may quickly be done (see Art. 6.2). It is believed that such critical conditions are rare. If he is interested in deflections at all, the engineer is usually concerned with those at the working load. Further, it is usually the case that only the floor beam deflections are of interest. On this basis the AISC handbook furnishes all the information needed to compute critical deflections.

RULE OF PRACTICE
Whenever current practice requires the computation of beam and girder deflections, then these will be made according to rules that follow (Art. 6.2). Otherwise no deflection computations are warranted. Methods are available for quickly computing frame deflections when a specific need exists.

6.2 COMPUTATION OF DEFLECTIONS

The deflection computations fall into two categories:

(a) Deflections at the full plastic load

(b) Deflections at working load.

Because of factors that are ignored in the analysis (such as residual stress, stress concentrations and strain-hardening) the actual magnitude of the deflection will be uncertain. However, the methods suggested will be sufficiently precise for practical purposes.

1. Deflection at Full Plastic Load

The approximate deflection at the inception of the full plastic load may be calculated by two methods:

(a) By the so-called "plastic hinge" method, a complete analysis is made of the load-deflection relationship up to the maximum load.(24,25)

(b) By a "Last Hinge" analysis, an estimate is made of full load deflections using the slope-deflection equations.(13)

Both methods are based on an idealized moment-curvature relationship shown in Figure 5. Each length of member retains its flexural rigidity, EI, except at the hinge sections. At these locations there is unlimited rotation at the M_p values.

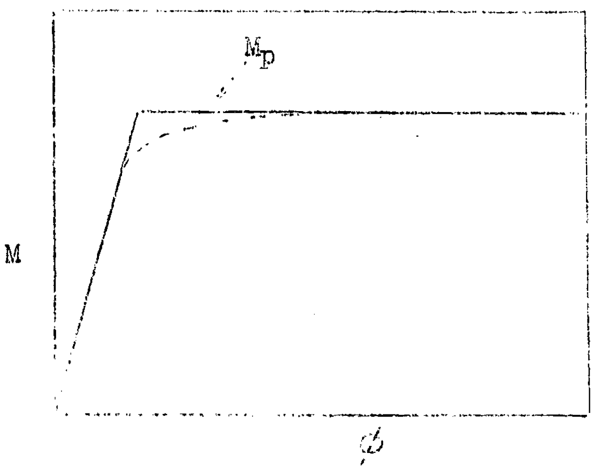


Fig.

The "Last Hinge" method is much more rapid and is preferable. It is based on the fact that just prior to the formation of the last plastic hinge, there is continuity at that point. Since the moments are everywhere known, then the slope-deflection equations may be used to solve for the relative deflection of segments of the frame.

The method is illustrated in Appendix 2 for a continuous beam and for a portal frame. The principal steps are:

1. Obtain the mechanism and the full plastic load. (This is part of the prior plastic analysis).

2. Compute the deflection of the various frame segments assuming, in turn, that each hinge is the last to form. (It is convenient to use the slope deflection equations).

3. Select as the correct deflection the largest value obtained (corresponding to the last plastic hinge).

For complicated structures it is desirable as a fourth step to make a check by assuming continuity at an arbitrary section. The deflection is computed, as are the "kinks" that form as a result of the (possibly) incorrect assumption as to which hinge is the last to form; the kinks are then removed by mechanism motion and the correct deflection obtained.

RULE OF PRACTICE

If the deflection at the full plastic load is required (see Art. 6.1) it may be computed by the "Last Hinge" analysis as outlined above.

---

2. Deflection at Working Load

The deflection of beams (as noted above) may be computed directly from the handbook if the end restraint conditions are known. This is based on the assumption that the beam is within the theoretical yield limit at working load. This is illustrated for the continuous beam example in Appendix 2.

The "Plastic Hinge" Method may be used to compute the load-deflection curve of the structure as the load increases from zero to a maximum value.[24,25] The method is particularly simple for beams and this is also illustrated in Appendix 2.

RULE OF PRACTICE

If the computation of beam deflections at working load is required, this may be done by reference to handbook tables. If frame deflections are required (and if the full load deflection is greater than the allowable deflection by an amount that exceeds the load factor of safety) then an elastic deflection analysis is necessary. In deciding whether or not computations will be made, the designer will use the same "yardsticks" as at present.

6.3 LIMITING VALUES OF DEFLECTION

Any rule of procedure suggested here must be tentative because no rational basis for deflection limits has as yet been established. A method has been established for selecting a proper working load for beams when either the deflection at working or at the full load is a limiting factor.[25] The latter is illustrated in Figure 6.

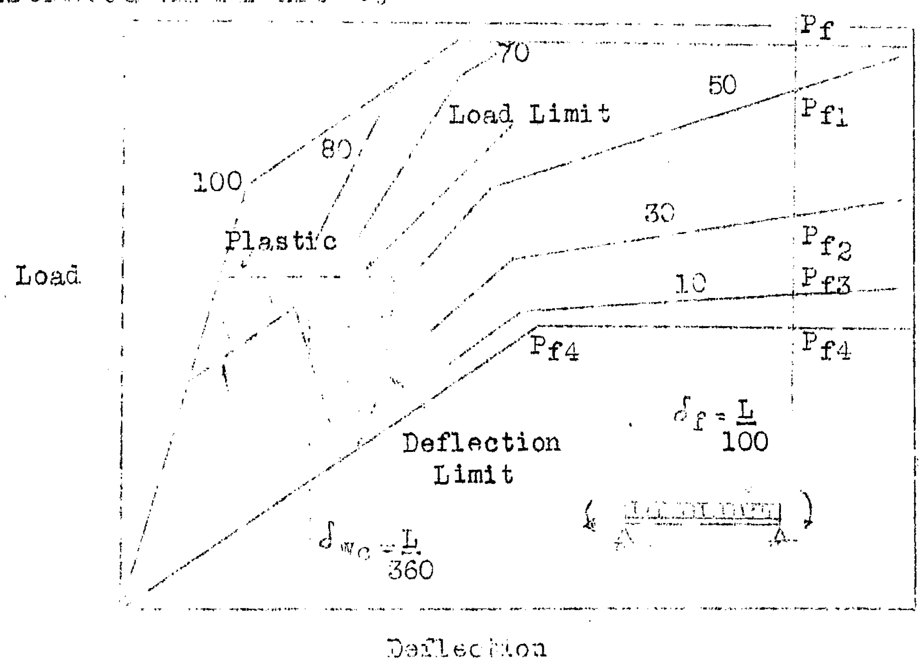


Fig. 6

RULE OF PRACTICE (Tentative)
The deflection of the structure shall not exceed that which would otherwise prevent the frame from performing its intended function. (Limit not yet set).

6.4 ROTATION CAPACITY

Rotation capacity, is the ability of a structural element to absorb rotations at near-maximum moment after reaching the hinge condition. It is usually R, expressed as the ratio of the average unit rotation, ϕ_A, to rotation corresponding to first yield, ϕ_y, or

$$R = \frac{\phi_A}{\phi_y},$$

the terms being defined in Figure 7.

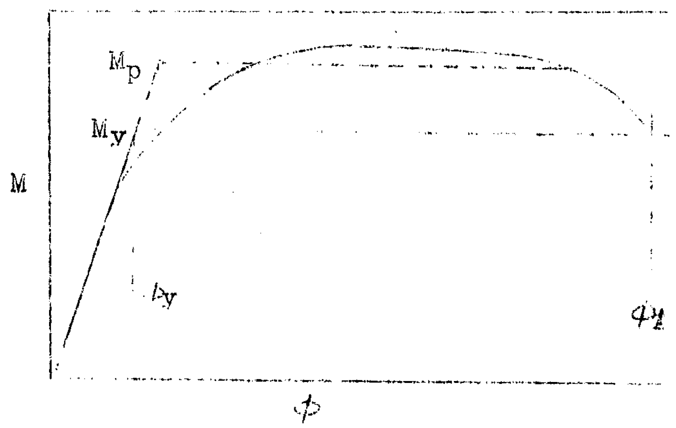


Fig. 7

Due to local buckling, lateral buckling, general instability, or fracture, a member or section may show only a small amount of plastic rotation at the hinge moment. However, if the structure is to support the predicted maximum load, M_p <u>must</u> be maintained at the first hinge while hinges are forming elsewhere.

Methods have been developed in the Lehigh Program for computing the required rotation capacity (it is dependent upon the loading and on structural dimensions). Normally, however, the designer will not have to make such computations, Rather the rules of practice will assure that structural joints contain sufficient rotation capacity.

RULE OF PRACTICE (Tentative)
All joints and sections will be proportioned to provide adequate rotation capacity.

FURTHER RESEARCH is necessary and is now underway at Lehigh to determine what the usual and the most severe requirements of rotation capacity will be. It is estimated that a value corresponding to the onset of strain-hardening ($R = 12$) will be sufficient.

7. CONNECTIONS

7.1 GENERAL REQUIREMENTS

The points of maximum moment (plastic hinges) usually occur at the connections. Thus proper performance of the connection is absolutely essential to the development of the predicted strength of the frame.

Fully welded continuous connections are the most conducive to the development of the greatest possible strength of a frame. Local fabrication conditions may dictate either welded "top-plate" connections or connections fabricated with high strength bolts. Where these may be shown by test to have dependable "plastic hinges" (even though of lesser magnitude than that of the section), then plastic methods will be applicable. Insofar as connections are concerned, however, these "rules" deal only with the fully continuous welded types.

There are four principal plastic design requirements for connections:

1. __Strength__ - The connection must be adequate to develop M_p of the members joined. (In a knee connection, only the weaker of the two will be developed.)

2. __Stiffness__ - Although it is not essential to the development of adequate strength (only the deflection is affected), it is desirable that the

average unit rotation not exceed that of an equivalent length of rolled beam.

3. Rotation Capacity - See Art. 6.4

4. Economy - This is particularly important since wasteful joint details may result in loss of over-all economy.

RULE OF PRACTICE

Connections shall be proportioned in such a manner as to develop the full plastic strength of the sections joined, with due consideration to joint stiffness when deflections are critical.

7.2 STRAIGHT CONNECTIONS (Portals)

None of the available wide-flange shapes have sufficient web thickness to prevent undesirable shear deformations when fabricated into a connection of the type shown in Fig. 8. (Some of the American Standard I's are satisfactory.) It is therefore necessary to stiffen the knee by doubling or by diagonal stiffeners. The following formula gives approximately the required web thickness, w_r, to assure adequate web strength:[26]

$$\boxed{w_r = 2.0\, S/d^2} \quad (7.1)$$

S is the section modulus and d is the depth of section.

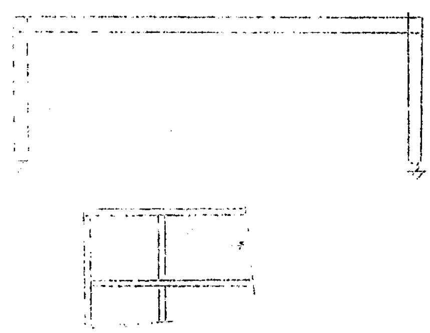


Fig. 8

This expression may be used to proportion a doubler as shown in Fig. 9. Alternatively, diagonal stiffeners may be used (Fig. 10). In this case, the following approximate equation may be used to determine the required thickness of the diagonal stiffener.[26]

$$\boxed{t_s = \frac{(w_r - w)\, d}{b_s\ \sqrt{2}}} \quad \cdot \quad \cdot \quad (7.2)$$

with the dimensions as shown in Figs. 9 and 10.

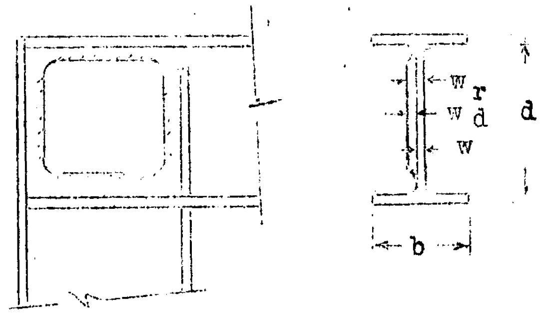


Fig. 9.

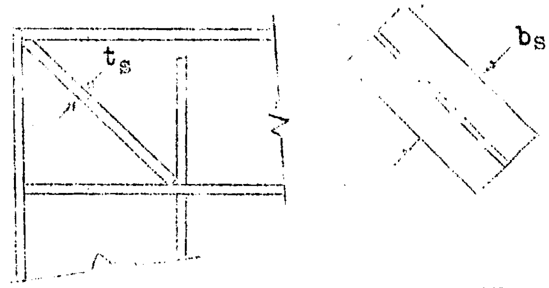


Fig. 10.

There are some advantages to permitting a certain amount of shear deformation in the knee web. If all the rotation occurs there, then the girder and column are spared severe plastic deformation of flange and web which may introduce local and lateral buckling.

A considerable number of tests have been made in which the diagonal stiffener thickness was made equal to that of the flange. Performance has been most satisfactory. In general this thickness has been greater than that required by Eq. 7.2. Some additional studies are needed to see whether or not these requirements are too restrictive, although the evidence suggests that they are not (see Fig. 3.6 of Ref. 37). Hendry has carried out tests on this topic[36].

For those applications in which the deflection of a frame at working load is critical, then the following rule for web thickness to assure sufficient knee stiffness is provisionally suggested,

$$\boxed{w_r = 2.6\ S/d^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.3)$$

Limits are yet to be established beyond which buckling will occur in unstiffened corner webs which are built up of plates and shapes (i.e. depth of section greater than 36"). For rolled shapes the provisions of this article will be safe and not uneconomical.

RULE OF PRACTICE (Tentative)
Straight connections of WF shapes will be stiffened with a diagonal stiffener (joining opposite corners) equal to the flange width and thickness. If the stiffener is objectionable, doublers may be added to give the required web thickness. Special consideration will be given to knees of built-up members of depth greater than that of rolled shapes.

FURTHER RESEARCH indicated above, includes a careful check on web stiffening, additional study into the problem of "tension" loading, rotation capacity of knees, and studies of large built-up connections. Knees on ships are of interest in this aspect of the problem.

7.3 TAPERED HAUNCHES

Haunched connections are a product of the elastic design concept by which the material is placed in conformity with the moment diagram. In plastic design, on the other hand, haunched connections are not required because lighter sections are possible by virtue of redistribution of moment. Thus the expense of fabricating the special haunches is eliminated. Although architectural considerations may demand the use of haunches, this does not require an elastic analysis. By assuming that the hinge sections are directly <u>adjacent</u> to the haunch and by suitable proportioning of the inner flange, a most satisfactory design is achieved.[26]

In order that the plastic moment of the beam be carried without extensive yielding within the haunch, it will probably be adequate to use an inner flange thickness determined from:

$$\boxed{t_f = 1.1\ Z/bd}$$

where t_f = haunch inner flange thickness, Z = plastic modulus of rolled shape, b = flange width, d = section depth, and the factor 1.10 is to account for the fact that the haunch plate is normally not parallel to the lower flange of the beam girder.

Design rules of the AISC[27] Nos. 1c, 4, and 8 shall be followed with regard to the other provisions.

<u>RULE OF PRACTICE</u> (Tentative)

Tapered haunches shall be proportioned such that flange yielding does not occur in the haunch, but that any possible hinges form outside (but adjacent to) the haunch. Increased thickness of the inner flange is required except for the corner bracket (45°) type. Lateral bracing shall be as specified in Chapter 12. Elsewhere, the applicable provisions of Ref. 27 shall be followed.

Certain additional research is appropriate into the proportioning of frames by plastic design using haunched connections

7.4 CURVED KNEES

> RULE OF PRACTICE (Tentative)
>
> The procedures outlined on p. 10 of Ref. 27 are to be followed, except that yielding is not to be allowed within the curved portion. See Art. 7.3.

7.5 BEAM-COLUMN CONNECTIONS

The numerous tests conducted in the Fritz Laboratory indicate clearly that where a fully continuous weld is placed on a direct welded beam-column connection, then plastic moments will be developed with no difficulty. The questions that remain are:

1. How much stiffening is required between column flanges for flange-connected beams?

2. Will biaxiality of loading, introduced in the case of connection types shown in Fig. 11, have any adverse effect upon connection behavior?

These and other questions are being examined in a program being resumed at Lehigh as a continuation of work done in 1954.

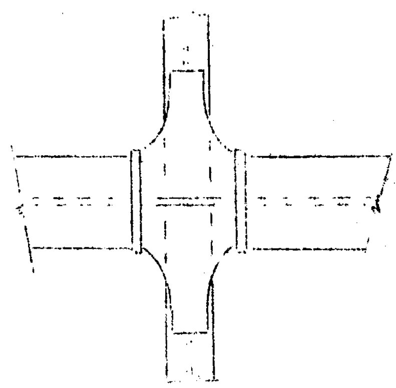


Fig. 11

Although it may be unduly wasteful of material, until the above tests are carried out it will be necessary to require full flange thickness of stiffener whenever the beam flange width approaches that of the column flange.

Top plate connections will be allowed when these are proportioned to develop the full strength of the connections joined.

> RULE OF PRACTICE (Tentative)
>
> Direct-welded connections shall be fabricated with full continuous welds. Beam-to-column-flange connections shall be backed up with stiffeners sufficient to transmit full flange stress. Top-plate connections must develop the full section strength and be sufficient to withstand reversed bending due to wind (if so assumed to act).

7.6 MISCELLANEOUS CONNECTIONS

1. Beam to Girder Connections

> RULE OF PRACTICE
>
> Subject to the provisions of Art. 5.2, continuous beam-to-girder connections of the type shown in the AISC and AWS Handbooks will be used. Wherever possible, fillet-type connections will be such that their strength over all will be greater than that of the member joined.

2. Purlins and Girts

> RULE OF PRACTICE
>
> Purlins and girts, designed for plastic action, shall either be proportioned with continuous welded connections or will be spliced for shear (where necessary) at a distance equal to L/6 from the roof beams, or from the columns.

3. Bracing

See Chapter 12.

4. Splices

The numerous beams that have been tested all indicate that a fully welded splice, either direct welded or with the aid of a splice plate will have adequate plastic characteristics.

Splices located intermediate between hinges may be designed for less than full moment capacity, still allowing the frame to carry its full load. This may lend some advantage to shop welding for full continuity with field connections having partial continuity.

RULE OF PRACTICE
Splices at points of maximum moment shall be adequate to develop plastic hinges. Direct welded types are suitable. Near points of inflection (Intermediate between adjacent hinges) splices may be designed for partial continuity.

8. COMPRESSION MEMBERS

8.1 CLASSIFICATION

Columns in continuous frames of the industrial type will almost always carry appreciable bending moment as well as direct force. As such they are termed "beam-columns" and will be discussed in greater detail in Chapter 11. This chapter is concerned with centrally-loaded columns only. Compression members under such a loading condition are seldom found in rigid frame construction.

Plastic design assumes that columns are stable under all conditions up to and at the full plastic load for the complete frame. Therefore, columns must be checked after the design is completed to see that they have adequate strength. (In the case of beam-columns it is also required that the members develop and maintain any plastic hinges assumed to form therein.)

Because of the critical nature of columns and their catastrophic type of collapse, economical design may eventually involve main plastic action only in bending and tension members, the columns being made sufficiently strong so that hinges develop elsewhere. Further work is needed here.

RULE OF PRACTICE

After the beam members have been selected, columns must be checked for adequate strength.

This elementary rule cannot be carried out in a completely rational manner until a large amount of research is completed. Column Research Council is leading in efforts to produce practical useful information regarding columns.[28]

8.2 CENTRALLY-LOADED COLUMNS

As a limiting possibility, the column carrying axial load without appreciable bending will now be considered. (It must be kept in mind in the case of any real column, as contrasted with an idealized "perfect" column, that there will always be some bending moment due to initial eccentricities or out-of-straightness)

Three types of load vs. longitudinal-deflection behavior may be distinguished, depending on the length of a column, as shown in Figure 12. A very short column will develop the yield point strength of the material and deflect longitudinally well into the strain-hardening range before buckling. It could be said that [illegible] column had reserve plastic strength. In the intermediate column lengths, yield may be very nearly developed, but rapid decrease in load carrying capacity with increasing longitudinal deflections will result as plastic buckling ensues. (Figure 12- b) In the case of a very long column that buckles elastically, longitudinal deflection may be developed without loss in axial load up to the moment that the stress due to axial load and bending moment, combined, reaches the yield level, when plastic buckling will occur. Numerous efforts have been [illegible] to utilize the behavior typified in "a" or "c" in the limit analysis of trusses (a subject beyond the scope of this report). Unfortunately, columns of the greatest practical interest (?? < L/r < 120) have negligible plastic reserve. Thus, for those cases in which the column is pin-ended or where the end moment conditions produce negligible effective eccentricities of load (and no restraint), then a formula for maximum strength (without special consideration of longitudinal deformation capacity) is what is needed.

The maximum strength of centrally-loaded steel columns is being studied in a current Column Research Council program.[13] The following formula is under consideration:

$$P/A = \sigma_y - 120\ L/r \quad \ldots\ldots(8.1)$$

(where the appropriate units are in psi and inches). The formula is based on a consideration of residual stresses.*

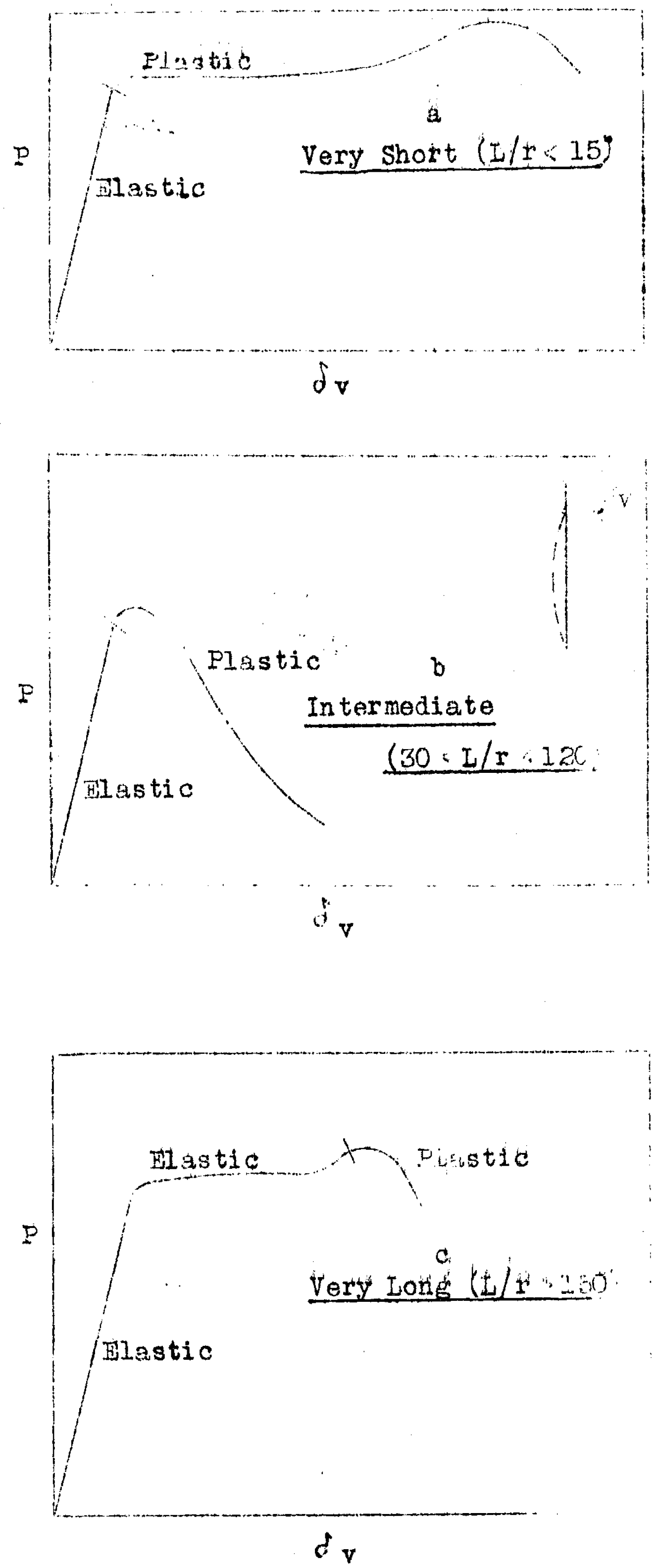


Fig.

Comparative Behavior of Various Length Steel Col.

* Additional precaution might be observed for these columns by using F = 1.85.

RULE OF PRACTICE (Tentative)

The load on a centrally-loaded column shall not exceed that given by the formula,

$$P/A = \sigma_y - 120\ L/r$$

8.3 COLUMNS IN TRUSSES

Although considerable research has been done on "limit analysis" of trusses, the subject of columns in trusses is beyond the intended scope of this report.

8.4 FRAMED COLUMNS

Considerable additional research remains to be done on framed columns. Some has been mentioned in the earlier articles, but most framed columns will be beam-columns (Chapter 11). As a special case, in tier buildings, if the beam sizes and the loads are symmetrical about the interior columns (no resultant end moments) and if provision is made for wind bracing with diagonals, it would seem proper to design for full (plastic) continuity of the beams, and to proportion interior columns for direct stress only. This would appear to place the design of the tier-type building on its simplest basis. Indeed, insofar as the beams are concerned, it is a return to the original design technique of KAZINCZY, (a technique that was used on buildings in Budapest in the early 1920's) which called for the proportioning of beams on the basis of "$\frac{wL^2}{16}$" (equilization of moment). (See Hoff's discussion of Ref. 1)

> **RULE OF PRACTICE** (Tentative)
>
> Where wind bracing in tier buildings consists of diagonals in wall panels, and where the framing and loading are such that there is no resultant end moment, then the interior columns may be checked on the basis that they must support only compressive load. (Eq. 8.1)

There *may* be cases in which centrally-loaded columns in rigid frames are also restrained against rotation at the ends. While this reduces the effective length, it is important to examine the adjoining beams which must provide this restraint. It has been shown for a simple example(31) that *greater* than simple beam bending moments result from some loading conditions. The overall economics must be considered--not just size of the columns.

In the case of elastic buckling it is well known that various end-restraint conditions may alter the effective pin-ended length of the column. The question arises as to whether or not these same length reduction coefficients can be used in determining the critical slenderness ratio that will determine whether or not a column buckles in the plastic range. In any case, it will be important to remember that the condition of *restraint* (not load) at the time of ultimate failure of a column determines the "K" factor by which the actual length of the column is multiplied to obtain the equivalent length. (Table I gives values of K for typical cases.)

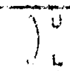
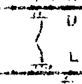
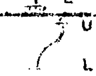
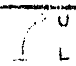
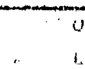
Type of Restraint	Buckled Form	Condition at U		Condition at L		Euler Length Factor "K"
		Rotation	Position	Rotation	Lateral Position	
1		free	fixed	free	fixed	1.0
2		free	fixed	fixed	fixed	0.7
3		fixed	fixed	fixed	fixed	0.5
4		fixed	free	fixed	fixed	1.0
5		free	free	fixed	fixed	2.0
6		Elastic Restraint	free	free	fixed	2.0 to ∞

Table I

Length-Reduction Factors For Columns

For example, a beam-column may be bent in double curvature by loads applied at each end. Thus, due to the applied loading, there would be an inflection point at the center of the column and at loads below the buckling load it might appear that the equivalent length of the column were one-half the total length. However, the column might actually have very little end restraint and as the buckling load were approached the double curvature would degenerate into single curvature with an equivalent length factor of one or thereabouts. It is important, therefore, to differentiate between deflected shape caused by applied loads and deflected shape inherently determined at near-buckling loads by the condition of end-restraint.

9. COMPRESSION DETAILS OF BEAMS AND COLUMNS

9.1 LOCAL INSTABILITY

In order to meet the requirements of strength and deformation capacity, the local compression (plate) elements of both beams and columns must have width-thickness ratios such that they will insure against premature plastic buckling after the yield point has been passed.

Standard rolled sections as listed in the AISC Handbook have been proportioned so that when used as columns or beams the yield point will be reached before local elastic buckling occurs. In the case of the more compact sections, with relatively thick flanges and webs, after the yield point is passed the section will continue to yield with little change in cross-sectional shape. However, in the case of lighter sections with relatively thin flange and web material, the web and/or flange elements may buckle plastically very soon after reaching the yield point stress. As a result of such plastic buckling, the hinge moment value may decrease as indicated in Figure 13 rather than approach the value of M_p. While such decrease might not be serious for simple beams, it would invalidate the assumption made in plastic theory for continuous structures that locations of maximum moment develop and

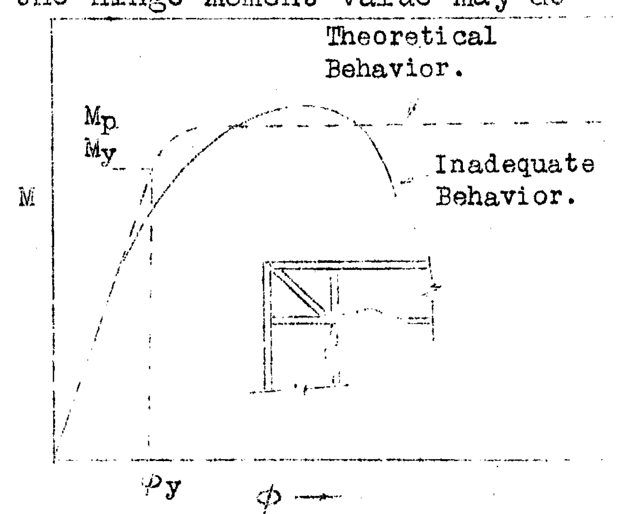


Fig. 13

maintain their full plastic moment strength while the structure deforms--and at increased load, additional plastic hinges develop at other points. The ability to deform locally as a "hinge" while maintaining the full plastic moment of resistance has been termed "rotation capacity". (Art. 6.4)

The problem, then, is to specify such proportions of cross-section (b/t and d/w) that the section will maintain its shape. It will probably be sufficient to require that

$$\epsilon_{max} \geq \epsilon_{st}$$

without loss of strength due to (plastic) local buckling. The problem is not separate and distinct from lateral buckling. In the inelastic range, both may occur and it is often difficult to determine which one is critical.

Although the complete theoretical solution has not yet been obtained, sufficient analysis and tests have been made so that a good idea can be had of proportions of unstiffened rolled sections that will be adequate.

On the basis of tests and additional theoretical analysis [52] (which is more precise than has been available heretofore), specifications tentatively may be suggested assuring that the shape will be able to deform to strain hardening without loss of strength due to plastic local buckling.

> **RULE OF PRACTICE** (Tentative)
>
> Local compression elements must be checked for adequate local buckling characteristics. Tentatively, it is required that the deformation capacity at full strength be equivalent to the onset of strain-hardening.

9.2 FLANGES AND WEBS

1. Compression Flange of Beam, Column Flanges

Referring to Figure 14 it is recommended that the width-thickness ratio, b/t, of the compression flange of a column be not greater than 17.(32)

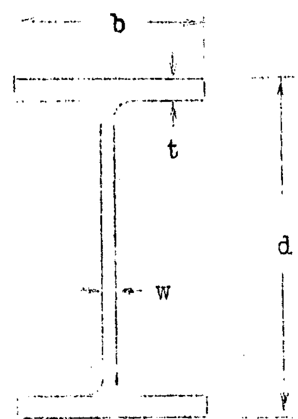


Fig. 14

All of the American Standard I-beam sections and most of the WF shapes that are used as beams will meet this flange thickness specification. In the case of WF sections normally used as columns, of about equal width and depth, the lighter weight sections will not meet this specification.

> **RULE OF PRACTICE** (Tentative)
>
> Compression flanges shall be proportioned with a width thickness (b/t) ratio no greater than 17.

2. Web Thickness

In the case of columns, the ratio of d/w shall be not more than 34. Practically all of the American Standard I-beam sections meet this specification and the WF sections normally used as beams meet this requirement in the case of the heavier sections of each depth for depths of 18 inches or under. With one or two exceptions, WF shapes 21 inches or more in depth do not meet this requirement. Most of the WF sections normally used as columns will meet the requirement if they also meet the flange requirement. Figure 34 (at end of paper) shows the range of b/t and d/w for beams and for columns.

It is evident from the results of tests that for beams and beam columns with low axial load, the above requirement is unnecessarily severe. It appears that a limiting value of d/w = 42 will be adequate. No results are as yet available, however, on specimens with larger d/w-ratios.

RULE OF PRACTICE (Tentative)

Webs of columns and beams carrying direct compression shall have a depth-to-web-thickness (d/w) ratio not greater than 34. (Figure 14.) Beams in bending may have d/w-ratios up to 42. The upper limit is not yet fixed.

9.3 STIFFENING

On the same basis as above, it is recommended that the width-thickness ratio of any compression or load-bearing stiffener be not more than 8.

The remedies for insufficient width-thickness ratios for columns and for beam-columns are to add stiffeners or to "box" the shape. In the case of beams under bending (with, perhaps small magnitude of axial thrust), additional lateral support will assist the shape in carrying its load. Possible stiffening devices are sketched in Figure 15. (It is not intended that they be used together, but to indicate alternate possibilities). Such devices are usually expensive, since they involve additional shop labor.

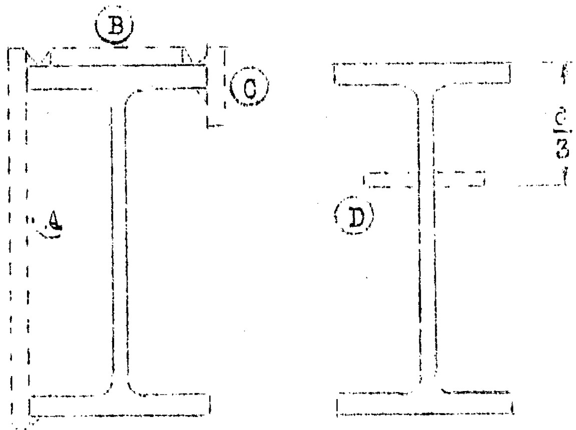


Fig. 15.

Another possibility is the use of box sections which have the additional advantages of higher shape factor and additional stability against lateral buckling.

When longitudinal stiffening of the type shown in Figure 15 (Type D) is to be used, the distance from the outer part of the compression flange to the compression stiffener should be about d/3.

When a compression stiffener as shown in Figure 15 (D) is used, the plastic hinge moment, M_p, will be slightly more than if the stiffener is not used. The section modulus, based on the maximum tension stress, will usually be slightly less than if the stiffener is not used. It is recommended that the stiffener be ignored for design purposes in the calculation of the plastic moment resistance. Longitudinal compression stiffeners, if required, should have equal or greater thickness than the web they reinforce, and should have a width not greater than 8 times their own thickness. When longitudinal stiffeners are used it is desirable that vertical load-bearing stiffeners be employed at reactions and locations of concentrated applied load.

> RULE OF PRACTICE (Tentative)
>
> Where proportions of rolled shapes do not meet the requirements of Art. 9.2, then stiffening shall be used to assure needed deformation capacity. A width-thickness ratio of 8 shall be used for load-bearing stiffeners.

9.4 MISCELLANEOUS DETAILS

In the design of details not covered herein, such as bearing value of webs at reaction points, end-connections, etc., the rules of design provided by the AISC "Specifications for the Design, Fabrication and Erection of Structural Steel for Buildings" may be followed, except that unit stresses permitted at conventional working loads should be multiplied by a factor of 1.65 to bring them approximately into line with the assumption of yield stress equal to 33,000 psi. The load used in these calculations would be the "full" load, to be divided by the factor of safety adopted for plastic design. Alternatively, details might be designed at working loads by conventional procedures with no change in allowable stress from values now used.

> RULE OF PRACTICE UNDER CONSIDERATION

10. BEAMS AND GIRDERS

10.1 BRACING

To prevent lateral buckling and consequent falling off of load resistance in the plastic range, frequent lateral support of beams as well as columns is necessary unless such beams and columns are an integral part of a strong wall or slab. This may be provided by means of the requirements of Chapter 12.

10.2 HINGE MOMENT

Assuming, now, that lateral buckling is prevented and that local buckling is eliminated by means of the requirements in Chapter 9, it is expected that "plastic hinges" with adequate "rotation capacity" will form at points of maximum moment thus permitting application of the procedures and theorems of plastic analysis of continuous frames. The next step is the evaluation of the hinge moment to be used as a basis for plastic design. For a rolled wide-flange beam, a typical theoretical curve of moment plotted against angle change per unit length of beam was shown in Figure 13. In Figure 35 (at end of paper) are plotted the average M-Ø curves for rolled beams and columns. The average value of the shape factor for all shapes (and for beams and columns taken separately) is 1.14, and this value might as well be used in design.

$$f = \frac{M_p}{M_y} = \frac{Z}{S}, \qquad \text{.................................(10.1)}$$

where Z equals the "plastic modulus" of the cross-section. In the case of symmetrical sections Z equals twice the static moment about the neutral axis of the half sectional area above or below the neutral axis. The AISC Steel Construction Manual lists the proporties of Tee-sections that are cut from rolled wide-flange shapes. For such tee-sections the location of the centroid is given. The plastic modulus equals the area of the rolled section multiplied by the distance between the centroid of the wide flange section and the centroid of the tee that is cut therefrom.

Wide-flange beams bent in the weak direction will not buckle laterally and, being essentially 2 rectangles, the plastic modulus, Z, will be approximately 1.5 times the section modulus, S.

Although it has been suggested in the past that the hinge moment be taken as a value M_o, being the average between M_y and M_p, (because of the effects of residuals, stress concentrations, the yielding process, shear, and deflections)[24, 25] it now seems an unnecessary complication. The tests on which the suggestion was made all involved considerable portions of the beams under pure bending. Under the more usual condition of a moment gradient, the hinge section will strain-harden. Thus the hinge moment should be taken at its full value,

$$M_p = \sigma_y Z$$

RULE OF PRACTICE

Where rolled beams are sufficiently braced laterally and when projecting elements have the proper b/t ratios to assure adequate rotation capacity, then the hinge moment shall be taken as $M_p = \sigma_y Z =$ 33,000 x 1.14 x S for bending about the strong (x-x) axis. A shape factor of 1.50 will be used for weak-axis bending. For built-up members, Z is twice the static moment of the cross-section.

10.3 SHEAR

Beam and girder design must include evaluation of shear strength and the possible reduction by shear of M_p should be avoided. As a reasonable approximation, general yield in the web of a WF beam may be expected when the average shear stress in the web equals one-half the yield stress in tension. The actual maximum shear stress in the web of a beam is greater than the average but the actual shear stress at initial yield is also somewhat greater than one-half the yield stress in tension. The proposed approximation agrees fairly well with tests [5]. The yield strength of the web material is likely to be slightly greater than that of the flange and an average shear yield stress level of 17,000 psi is suggested for plastic design purposes. The total beam "shear yield" value would then be:

$$V_y = 17{,}000\ wd$$

In certain loading cases [1] yielding may be expected at loads considerably less than those given by the above equation. Tests have shown that this is critical only if deflections of the structure are of over-riding importance.

Theoretically, a beam may "collapse" due to combined effects of bending and shear forces. This may be expressed as an influence of shear on the moment capacity. Theoretically, the influence is negligible unless the distance between adjacent hinges is less than eight times the depth of the member; even when this distance is as short as four times the depth, the influence is only 10%.

Certain tests are now underway to examine critically the provisions of this article.

> RULE OF PRACTICE (Tentative)
>
> The maximum allowable shear in a beam is to be obtained from V = 17,000 wd. The full plastic moment may be assumed unless the distance between adjacent hinges is less than about 6d.

10.4 CROSS-SECTIONAL FORM

It is recommended that use be made of sections that are symmetrical about their principal axes. The use of longitudinal compression stiffeners is not considered as making the section unsymmetrical since it is not included in calculating the bending resistance.

> RULE OF PRACTICE
>
> Sections in which plastic hinges are expected to form shall be symmetrical about their principal axes. Unsymmetrical sections may be used if sufficiently braced and if they are demonstrated to have adequate strength and deformation characteristics.

11. BEAM-COLUMNS

11.1 MODIFIED HINGE MOMENT

The "pure" column under axial load alone will rarely be encountered in a continuous frame. The usual case will be that of a "beam-column" in which bending frequently will be the primary cause of failure rather than direct load.

Considering the members as being designed primarily to resist bending moment, direct stress has a two-fold influence on behavior. The first is to change the magnitude of the plastic hinge moment and the second is to influence the curvature.

In Figure 16 is shown the "interaction" curve for a rectangular cross-section (approximately equal to WF shape bent about the weak axis). The equations for initial yield (M_{yc}) and modified hinge moment (M_{pc}) are given by

$$\frac{M_{yc}}{M_p} = \frac{2}{3}\left(1 - \frac{P}{P_y}\right)$$

$$\frac{M_{pc}}{M_p} = 1 - \left(\frac{P}{P_y}\right)^2 \quad \ldots\ldots(11.1)$$

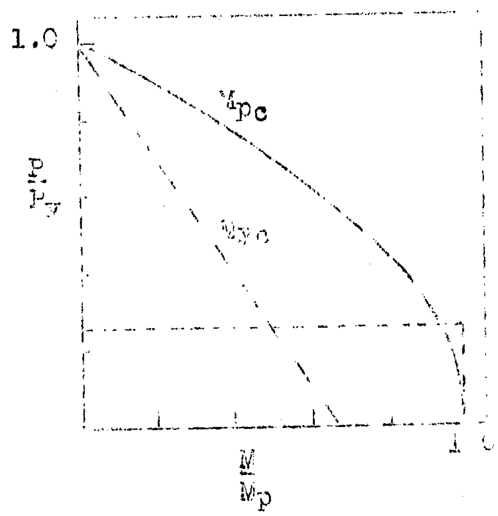


Fig. 16.

With small error (5%) the effect of direct stress may be neglected when the axial load is less than 25% of the full yield load ($P_y = \sigma_y A$). See Figure 16. For P greater than this limit, Eq. 11.1 may be used.

The interaction curve for a typical WF Shape bent about the strong axis is shown in Figure 17. With an error of about 5%, axial load can be neglected up to $P/P_y = 0.15$. The "exact" formula for the M_{pc} curve is somewhat involved; for $P = 0.15 P_y$ it is approximated by the equation

$$\frac{M_{pc}}{M_p} = 1.16\,(1 - P/P_y) \ldots\ldots (11.2)$$

which is also plotted in Figure 17.

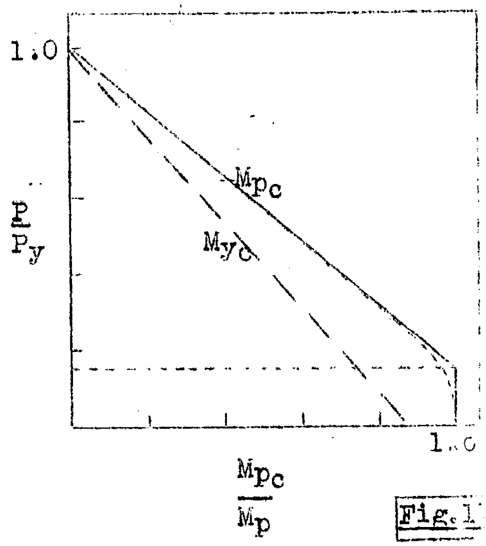


Fig. 1

Use of Eq. 11.2 (or Eq. 11.1) in design would be after the plastic analysis has been completed, the shape selected, and the direct loads computed. If P/P_y is greater than 15%, then the dependable hinge moment is computed from Eq. 11.2 (or 11.1) and a new shape is selected by multiplying the original section moulus by the ratio $\frac{M_p}{M_{pc}}$.

The 5% reductions computed by the above two equations are undoubtedly offset by strain-hardening, an effect normally neglected in plastic analysis.

RULE OF PRACTICE

The hinge moment should be assumed equal to M_p unless the axial load is greater than 15% of the full yield load. In the latter case the hinge moment may be computed from Eq. 11.2. The section modulus to use is then determined by multiplying the value of S found from the preliminary analysis by the ratio $\frac{M_p}{M_{pc}}$ using the expression

$$\frac{M_p}{M_{pc}} = \frac{.85}{(1 - P/Py)}$$

11.2 THE M-P-Ø RELATIONSHIP

The second influence of axial load on the behavior of a beam is its influence on the curvature. A recent Lehigh report[6] gives methods and equations for determining the elastic and plastic moment-curvature relationship of WF shapes when used as beam-columns. The solution includes the effect of residual stresses. Ref. 7 also discusses the interaction curve mentioned in Art. 11.1.

If the M-Ø curve for a beam-column is needed, the linear relationship for M and Ø holds up to M_{yc},

$$\phi_{yc} = \frac{M_{yc}}{EI} \quad \text{................(11.3)}$$

Figure 18 is typical of the M-Ø curve for various magnitudes of axial thrust. The upper limit of each curve is determined approximately from Eq. 11.2.

Normally, the designer will not be directly concerned with these curves. They are a basic tool for the determination of column strength,[6] but tables, charts or curves would summarize theoretical analysis in suitable design form.

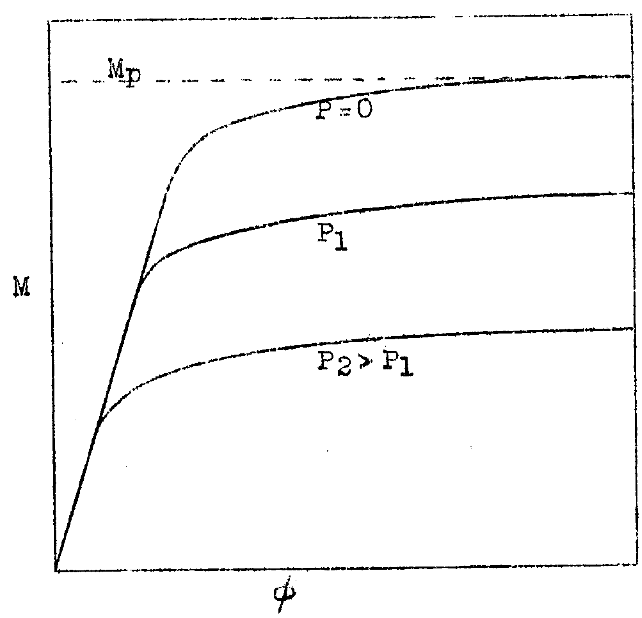


Fig.18

11.3 BEAM-COLUMN STRENGTH

It is desirable to set up a criterion to determine in what range of beam-column slenderness one may apply the assumptions of plastic hinge analysis, i.e., hinge moments maintained during deformations carried beyond the yield range. Any criteria must be tentative because at the present time only a beginning has been made in the theoretical solution to the lateral-torsional buckling problem; this is involved in the collapse of many columns bent about their strong axes. Only just recently are rational solutions becoming available enabling one to predict with accuracy the maximum plastic strength of columns without side-sway and with various conditions of end restraint and loading.(6, 33)

In many industrial frames the axial force is low. All tests have shown that the effect of direct stress may be neglected when P/P_y is less than 10% (except for the very unusual single-curvature case). It has been suggested (Chapter 8) that in tier buildings, when side-sway is prevented by diagonal panel bracing, the interior columns might be designed as compression members so long as certain restrictive boundary conditions are met.

Thus the various critical cases appear to be in the exterior columns of panel-braced tier buildings, in interior columns with effective eccentricity of load, in columns of knee-braced tier buildings, and in those industrial buildings in which column loads are relatively high.

A tentative rule is now being examined; although a few test results seem to support its applicability it is not sufficiently confirmed to be put forward at this time. It consists of a formula of the interaction type:

$$\left(\frac{M}{M_p}\right)\left(\frac{K}{C_1}\right)\left(\frac{Ld}{bt}\right) + \left(\frac{P}{P_{cr}}\right)\left(\frac{K}{C_2}\right)\left(\frac{L}{r}\right) \leqq 1$$

L = actual overall length of the member

K = length modification factor dependent on the distribution of force and type of restraint

The constants C_1 and C_2 would be selected in such a way that in the limiting condition of zero axial thrust the section would carry the full plastic moment and the first term in the above equation would equal 1. Alternatively, when the moment was zero, the constant C_2 would be selected in such a way that the column would carry the allowable centrally-applied load (and the second term would be unity).

Considerable additional study is needed, as are tests which introduce side-sway.

RULE OF PRACTICE (Tentative)

If the direct stress is low ($P/P_y < 15\%$) and unless the member is bent in single curvature (an unusual condition), direct stress may be neglected in industrial (one-story) frames. The provisions of Art. 8.4 will be followed for the interior columns of panel-braced tier buildings in those symmetrical cases in which there is no resultant end moment. All other cases require special treatment, and the subject is under consideration.

12. LATERAL BRACING

The function of bracing is to provide lateral support to the beam, column and, in particular to the connections to assure that they will not buckle sidewise either due to lateral buckling or to lateral buckling brought on by local buckling. Proper bracing is the best means for assuring adequate rotation capacity (Art. 6.4).

Where the structure is enclosed by walls or slabs normal to the plane of the frame, as in ordinary fireproof construction, then it is assumed that the enclosing material provides adequate lateral support. Thus the problem is applicable to industrial building frames or other "open" types of construction.

Numerous measurements have been made in laboratory tests of the forces necessary to provide lateral support,[26,34] and the magnitude has never exceeded 1% of the product of yield-point stress and cross-sectional area of the member being braced. It is important, however, that adequate stiffness be supplied; therefore, the lateral support devices should not be allowed to reach yield stress at any time. On this basis, a rule may be formulated for cross-sectional area of lateral supports (ties assumed normal to member being supported). The "1%" factor is tentatively doubled because field conditions may not be well-defined.

$$A_b = .02 \frac{\sigma_y}{\sigma_w} A$$

where

A_b = area of bracing material

A = area of member being braced

σ_y = yield stress of member being braced (33 ksi)

σ_w = allowable stress in bracing member (20 ksi) preferably a tension member,

thus, approximately,

$$\boxed{A_b = .04A} \quad \text{- - - - - - - - - - - (12.1)}$$

In addition, to provide rigidity, the members should be as short as practicable.

The location of the bracing is just as important as its strength and stiffness. In columns, they should be located near expected hinge locations, particularly on the compression flange. At connections, they should be located at the points at which yielding is expected,[26] and consistent with with the previous rules, these points would be as shown in Figure 19 (note that support at points A and C is just as effective as the single support at B.)

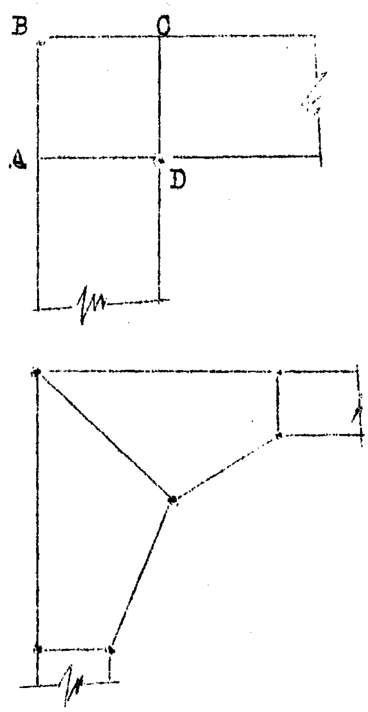


Fig. 19

A point of present uncertainty is the frequency of location of lateral support points along a beam. The British use an L/r ratio of 100 as a provisional spacing of lateral support(8). It seems that a specification ought to be based on the resistance to twist, and the Ld/bt parameter might be more appropriate.

In the absence of adequate analytical or test information on the lateral plastic buckling of short beams, one approach is to specify a ratio of Ld/bt of not more than 100 for beams, arrived at on an arbitrary basis by comparison with the column analysis. A "perfect" steel column will begin to buckle elastically instead of plastically when the slenderness (KL/r) ratio is more than about 90--and according to the AISC beam formula, lateral (torsional) buckling of a perfect beam will occur elastically instead of plastically when KLd/bt is more than 600. Now, a centrally-loaded column will carry its full plastic load while the fibres deform up to strain-hardening so long as KL/r is less than about 15 (Fig. 12). Therefore it might be presumed that a beam of KLd/bt less than 100 will not buckle plastically soon after the yield point is reached. The reasoning here is tenuous, but there is a limited amount of experimental evidence to indicate that this value is at least conservative and that revisions upward might become desirable when more study has been given to this important problem. Load distribution and, as in the case of a column, end or intermediate restraint conditions may be introduced to determine an "equivalent" beam length. Lateral supports <u>intermediate</u> between the ends should usually be considered as hinged. Similarly, an assumption of K = 1 should probably be made for one span of a continuous beam of equal spans.

An alternate approach may give an answer that will be sufficient for many cases. Since the loss of stability is aggravated by plastic action, additional lateral support might be provided in a girder at those points at which plastic hinges are expected, the elastic lateral support rules being used elsewhere. It is planned to conduct some experiments on this basis.

It should be kept in mind that these bracing requirements are necessary only to assure that the maximum peak overload be carried by the frame. Actually the forces to maintain the member in the straight position are negligible until after the hinges have formed[26,34].

RULE OF PRACTICE (Tentative)

Sufficient lateral support shall be provided in order that lateral buckling will be prevented. The cross-sectional area of bracing members (normal component) shall be no less than 4% of the area of the member being braced and shall be as short as practicable in length. Points of lateral support in connections shall be at the points of expected hinge location and both inner and outer flanges of the member shall be braced. (Specific spacing of lateral support to beams and girders is not yet specified. Support at the hinge locations plus those required elastically should be sufficient).

FURTHER RESEARCH is required, to cover,

(a) a check of 4%-area rule

(b) examination of practical means of designing lateral support members.

(c) study of spacing of lateral support members in girders and in columns.

13. SUMMARY

The recommendations contained in this report are applicable to continuous structures such as industrial frames, tier buildings, and similar military structures. Consideration of trusses has been excluded.

Each of the problems or factors that could be anticipated up to the present time has been described, the status of solution has been given, and a "Rule of Practice" has been suggested. Only about half of the rules are tentative or "Under Consideration", and in many of these, conservative provisions are indicated.

Although the following listing is not a review of all the articles, a scanning of the report indicates that the "Rules" permit significant application of plastic analysis in structural design. In other words, the provisions of the rules do not constitute undue limitations.

1. Types of Construction cover a range that is sufficient to assure extensive application. This is particularly true in the case of industrial frames and continuous beams.

2. Methods of Analysis and Design Procedures offer distinct advantages over conventional methods.

3. Loading may be considered as proportional (or "static") so frequently, that the majority of cases will be covered.

4. The tentative Load Factor of Safety represents a value that allows the realization of the economy being sought.

5. The provisions concerning Deflections present no more of a restriction to plastic design than they do to conventional procedures.

6. Connections are suitable for a large range of shapes, and numerous economic types meet the requirements.

7. Although the formula for centrally-loaded Columns offers but modest economy, the fact that axial load may be entirely neglected for a considerable number of Beam-Columns is of real advantage. Specific recommendations for beam-columns supporting large axial loads are not included, but it is likely that any procedure proposed will not be complex and will offer economies over present methods.

8. The rules for the ratios of compression details to prevent Local Buckling allow the use of most of the available WF shapes without extra stiffening.

9. Rolled WF and I-shapes develop suitable hinge moments, and modification due to the presence of Shear would only be required in unusual cases.

10. The provisions for Lateral Bracing present no serious difficulty insofar as size of member is concerned. There is a real problem in specifying the spacing of lateral supports to beams. Tentative solutions are now being studied.

Except for the uncertainty of bracing provisions, none of the rules prevent direct application of plastic design to structural problems. Some of the additional research is desirable to broaden, even further, the applicability of the plastic methods, and some would be just as useful to conventional as to plastic design. It is considered that immediate application to the design of certain rigid structures such as continuous beams and industrial frames is appropriate. Indeed, it is already under way.

14. ACKNOWLEDGEMENTS

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Project 206s-1-F

16. NOMENCLATURE AND TERMINOLOGY

A Area of cross-section

b Flange width

d Depth of section

E Young's modulus of elasticity

E_{st} Strain-hardening modulus

E_t Tangent-modulus

F Load factor of safety

f Shape factor $= \frac{M_p}{M_y} = \frac{Z}{S}$

G Modulus of elasticity in shear

I Moment of inertia

KL Effective (pin-end) length of column. K = Euler length factor

L Span length. Actual column length.

M Moment

M_p Full plastic moment

M_{pc} Plastic hinge moment modified to include the effect of axial compression

M_y Moment at which yield point is reached in flexure

M_{yc} Moment at which initial outer fibre yield occurs when axial thrust is present

P Concentrated load

p Distributed load per unit of length

P_{cr} Useful column load. A load used as the "maximum column load"

P_f Full load (working load x load factor of safety)

P_p Full plastic load on a structure computed by simple plastic theory

P_s Stabilizing ("shakedown") load

P_y Axial load corresponding to yield stress level ($\sigma_y A$)

R Rotation capacity

r Radius of gyration

S Section modulus

t Flange thickness

V Shear force

w Web thickness

Z Plastic modulus

δ Deflection

ϵ Strain

ϵ_{st} Strain at strain-hardening

θ Measured angle change; rotation

μ Poisson's ratio

σ Normal stress

σ_y Yield stress level

τ Shear stress

ϕ Rotation per unit length, or average unit rotation; curvature

APPENDIX 1

PLASTIC ANALYSIS

1. RESERVE IN STRENGTH DUE TO FORMATION OF PLASTIC HINGES

The fixed-ended uniformly-loaded beam of Figure 20 will be used to illustrate how pleastic hinges allow a structure to deform under load beyond the elastic limit, permit a redistribution of moment and, thereby, an increase in load capacity.

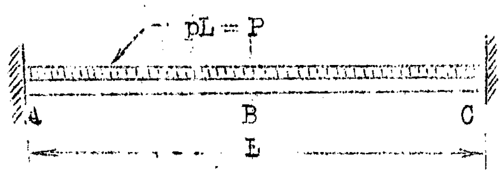


Fig. 20

By an elastic analysis, the deflection curve of Figure 21 and the moment diagram of Figure 22 could be determined when yielding first commences. This is indicated in the figures as Phase 1. The center moment is $\frac{pL^2}{24}$ and the end moment is $\frac{pL^2}{12}$.

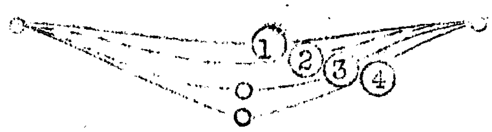


Fig. 21

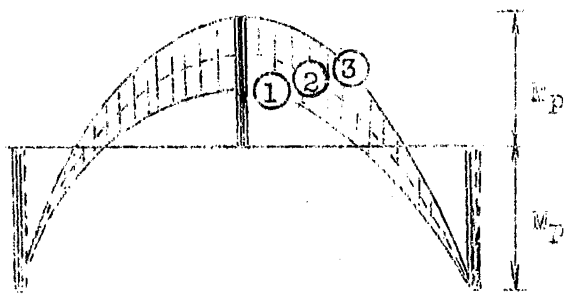


Fig. 22

Since the moment at the ends is at the yield point, "hinge action" will start at this point when the beam is deformed with more load. The plastic hinge is described diagramatically by the curve of Figure 23 in which moment,

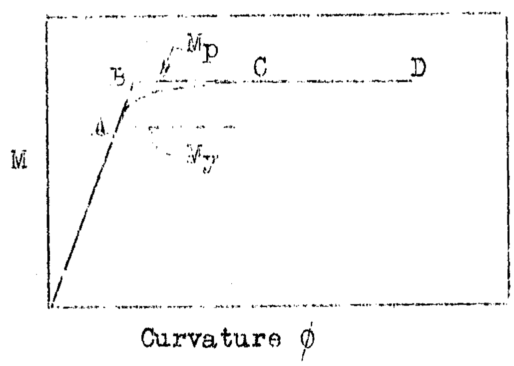


Fig. 23

M is plotted against the curvature, Ø. (Although the actual M-Ø curve is similar to OACD, the flexural analysis is simplified by considering it to be OBD--and the resulting error is small). Thus, if Fig.24(a) is used to represent the M-Ø action at Points A and C of the beam, while Fig.24(b) is used for Point B, the corresponding moments for Phase 1 are shown at $M = M_p$ and $M = \frac{1}{2} M_p$, respectively.

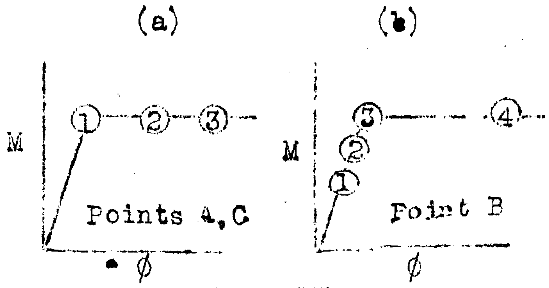


Fig.24

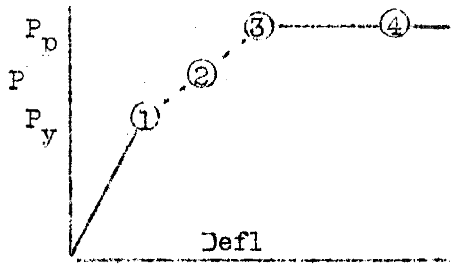


Fig.25

As load increases, the beam now behaves as if it were simply-supported, except that the end moment remains constant at M_p. The center of the beam still has available moment capacity, and thus an intermediate Phase might be as shown by "2". As shown in Fig.25, the deflection increases at somewhat faster rate (the "rate", in fact, is the same as that of a simply-supported beam of length L).

At Phase 3 the beam will have reached its maximum load, since the moment capacity at the beam center is exhausted (Fig.22, 24b). Definite "kinks" will have formed at the ends (Fig.21) due to rotation at constant moment. Beyond this, Phase 4, the beam deforms as a mechanism under constant load (Fig.25).

The increase of load between initial yield and the maximum plastic strength (full load) is represented by the shaded portion of Fig. 22. The yield load is given by

$$\frac{P_y L}{8} = \frac{3}{2} M_y$$

The full load is given by the moment diagram of Fig. 22 at full plastic load as,

$$\frac{P_f L}{8} = 2 M_p$$

For $\frac{M_p}{M_y} = 1.14$, then the ratio between the full load and the yield load is

$$\frac{P_f}{P_y} = \frac{2M_p}{3/2\ M_y} = \frac{4}{3} \times 1.14 = 1.52$$

That is, the ratio of the load capacity of the beam to the load at initial yield is about 50%.

Two methods of plastic analysis will now be described.

2. EQUILIBRIUM METHOD (SEMI-GRAPHICAL)

There are three conditions that must be satisfied in any plastic analysis:

1. Every part of the structure must be in equilibrium.
2. A mechanism must be formed in the sense that a small motion occurs without increase in load.
3. At no point may the moment be greater than the plastic moment, M_p.

For a relatively simple structure like that shown in Fig. 20, the correct solution may be arrived at very quickly. To form a mechanism there must be three plastic hinges and these must be at the "maximum" points A, B, and C. Their value is M_p. To be in equilibrium $\frac{pL^2}{8} = 2\ M_p$, and the maximum load has been determined.

The method, in general, consists of constructing the moment diagram disregarding the redundants, followed by the construction of redundant moment diagrams in such a way as to allow hinges to form. In more involved structures it is necessary to guess where these hinges will be located and to make several trials. Since the method has been described completely in Ref. 8, and since numerous examples have been given there, further discussion here is unnecessary.

3. VIRTUAL DISPLACEMENT METHOD

Since the method has been completely described in numerous references(11,13) only a brief outline is given here, and is based, particularly, on Ref. 11.

The method consists of examining the frame to see the different ways in which mechanisms can form by the development of plastic hinges. To each possible mechanism there corresponds a full plastic load, and this may quickly be determined by use of the principle of Virtual Displacements. It has been shown(14) that the actual or correct mechanism for the given frame is the one that gives the smallest value of the full load.

Thus, the problem is to find all the possible mechanisms and select the one that gives the lowest full load value.

Fig. 26 will serve as an example, although it is recognized that the selection of span-height ratio, the fixing of the bases and the ratio of vertical to side load makes the problem somewhat unrealistic. The section is assumed constant throughout and the problem is to find the correct value for the full load P.

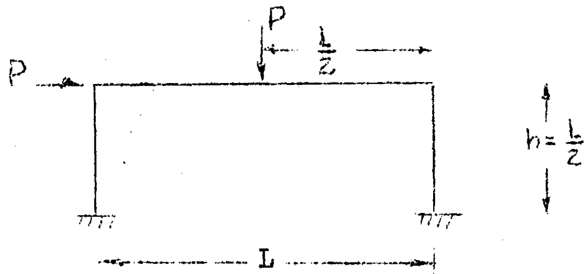
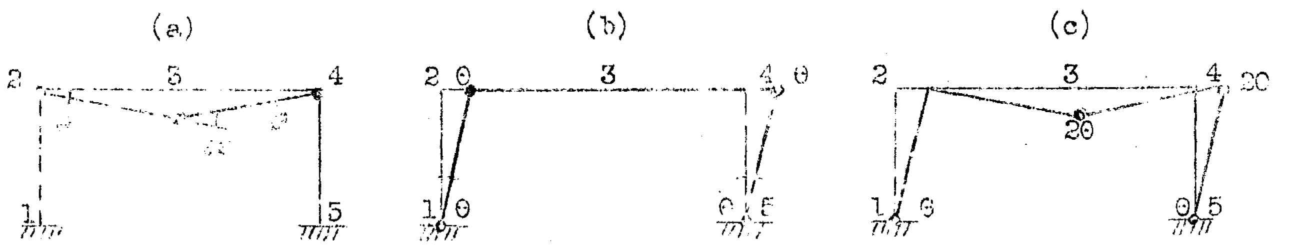


Fig. 26

The possible mechanisms are shown in Fig. 27. The virtual displacement principle (external work done by loads equals internal work done at hinges during a small displacement), may be used to find the full load for each mechanism. These



MECHANISMS - HINGE DIAGRAMS

$$P\frac{L\theta}{2} = M_p(\theta+2\theta+\theta) \qquad P\frac{L\theta}{2} = M_p(\theta+\theta+\theta+\theta) \qquad P\frac{L\theta}{2}+\frac{L\theta}{2} = M_p(\theta+2\theta+2\theta+\theta)$$

$$P = \frac{8M_p}{L} \qquad P = \frac{3M_p}{L} \qquad P = \frac{6M_p}{L}$$

VIRTUAL DISPLACEMENT EQUATIONS

Fig. 27

equations are shown in Fig 27 beneath each mechanism. The smallest load corresponds to the mechanism of Fig 27-c.

It is desirable to make a static check by drawing the moment diagram. This is done in Fig. 28 and is accomplished by laying off the Moments at 1, 3, 4, and 5 equal to M_p. The moment at 2 is determined by equilibrium.

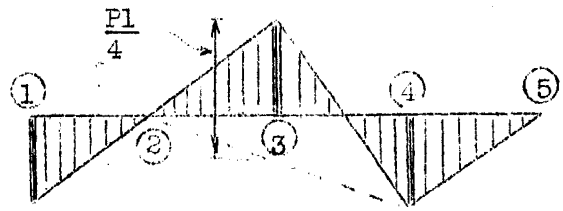


Fig. 28

Since the moment is nowhere greater than M_p the solution is correct. The three general conditions (mechanism, equilibrium, and $M = M_p$) have been satisfied.

In solving problems for more complex frames it is necessary to follow a systematic procedure since the different mechanisms will not be obvious. There are four steps that must be followed:

1. Determine the various possible elementary mechanisms.

2. Make such combinations of elementary mechanisms as will reduce the full load, P_p.

3. Compute the full plastic load for each case.

4. Selecting the lowest value of the load (or maximum M_p value) check the "plasticity" condition by equilibrium, constructing the moment diagram.

Each of these steps will now be discussed more fully and general rules will be outlined.

(3a) Elementary Mechanisms

There are three types of elementary mechanisms,

(1) Beam Mechanisms

(2) Panel Mechanisms

(3) Joint Mechanisms

Types (1) and (2) are illustrated in Figs. 27a and 27b, respectively. Type (3) is encountered whenever three or more members join at a connection.

These elementary mechanisms correspond to equations of equilibrium. In a complicated structure it is possible to determine, in advance, the number of independent elementary mechanisms by the following simple rule:

Let

X = Number of possible plastic hinges

Y = Number of redundants

Z = Number of elementary mechanisms

Then

$$Z = X - Y$$

Thus in the example,

Number of possible plastic hinges (at connections and under loads)	=	5
Number of redundants	=	3
Number of independent mechanisms	=	2

and these are the two mechanisms of Fig. 27a and 27b.

In the case of distributed loads, as a first approximation (good within about 5%), the hinge location can be assumed at the center of the span on which the distributed load acts.

(3b) Combinations of Mechanisms

The general rule is to make such combinations that one or more plastic hinges present in the two elementary mechanisms disappear. Thus in the example, if Mechanism (a) is combined with Mechanism (b), a new one is obtained (c) in which there is no hinge at Joint 2.

By sketching the frame in the deformed condition, one may determine the relative rotations at each joint and write down the virtual displacement equations directly.

(3c) Computation of Full Load and Check of Plastic Moment Condition

When it is believed that the correct answer for the full load has been obtained (lowest value of the full plastic load) then a bending moment diagram may be drawn and the moment at every joint computed by the equations of static equilibrium. If the moment at every section is nowhere greater than the M_p-value of the member, then the answer is the correct one.

* * *

This brief account should serve to describe the method. Certain details simplifying the calculations for gabled roof frames are contained in a more detailed description of plastic analysis methods now in preparation[15]

2. DEFLECTION OF A FRAME AT FULL PLASTIC LOAD

Example: Frame of Fig 31 (same as Appendix 1)

Mechanism and Full Plastic Load (Fig 31-b)

From Appendix 1,

$$P_p = \frac{6 M_p}{L}$$

Free-Body Diagrams: Fig 31-c

Computation of Vertical Defl.

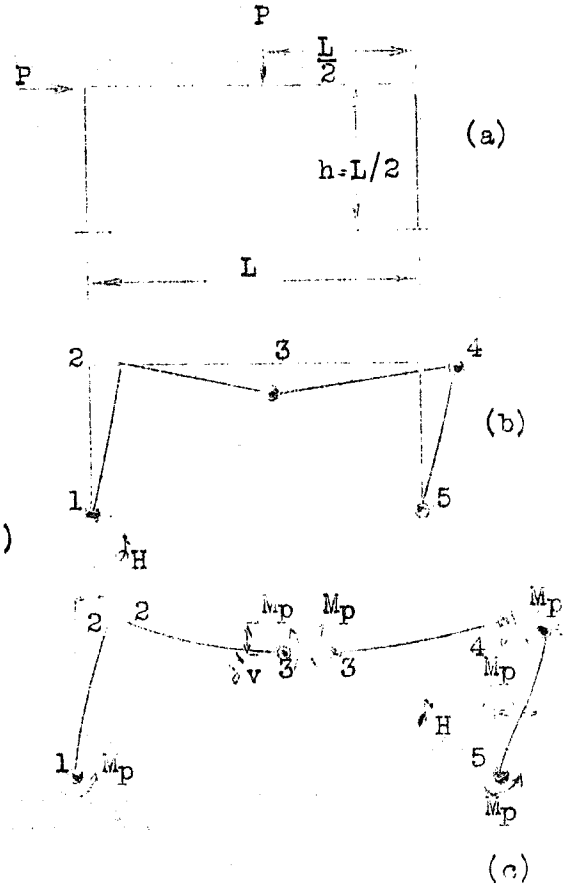


Fig.31

$\boxed{\text{RATIO OF } \delta_H \text{ AND } \delta_V}$: Continuity at Section 2, ($\theta_{23}=\theta_{21}$)

$$\theta_A = \theta_A' + \frac{\Delta}{\ell} + \frac{\ell}{3EI}\left(M_{AB} - \frac{M_{BA}}{2}\right)$$

$$\theta_{23} = 0 + \frac{\delta_{V2}}{L/2} + \frac{L/2}{3EI}\left(0 + \frac{M_p}{2}\right)$$

$$\theta_{23} = \frac{2\delta_{V2}}{L} + \frac{M_p L}{12EI}$$

$$\theta_{21} = 0 + \frac{\delta_{H2}}{L/2} + \frac{L/2}{3EI}\left(0 + \frac{M_p}{2}\right)$$

$$\theta_{21} = \frac{2\delta_{H2}}{L} + \frac{M_p L}{12EI}$$

$$\frac{2\delta_{V2}}{L} + \frac{M_p L}{12EI} = \frac{2\delta_{H2}}{L} + \frac{M_p L}{12EI}$$

$$\boxed{\delta_V = \delta_H}$$

$\boxed{\text{TRIAL AT SECTION 1}}$: (Member 1-2, $\theta_1 = 0$)

$$0 = 0 + \frac{\delta_{H1}}{L/2} + \frac{L/2}{3EI}(-M_p + 0)$$

$$\delta_{H1} = + \frac{M_p L^2}{12EI}$$

$$\boxed{\delta_{V1} = \frac{M_p L^2}{12EI}}$$

$\boxed{\text{TRIAL AT SECTION 3}}$: $\theta_{32} = \theta_{34}$

$$\theta_{32} = 0 + \frac{\delta_{V3}}{L/2} + \frac{L/2}{3EI}(-M_p + 0) = \frac{2\delta_{V3}}{L} - \frac{M_p L}{6EI}$$

$$\theta_{34} = 0 - \frac{\delta_{V3}}{L/2} + \frac{L/2}{3EI}\left(M_p - \frac{M_p}{2}\right) = -\frac{2\delta_{V3}}{L} + \frac{M_p L}{12EI}$$

$$\theta_{32} = \theta_{34}$$

$$\boxed{\delta_{V3} = \frac{M_p L^2}{16EI}}$$

TRIAL AT SECTION 4 : Similar procedure using $\Theta_{43}=\Theta_{45}$

$$\delta_{V4} = \frac{M_p L^2}{24EI}$$

TRIAL AT SECTION 5 : Similar procedure using $\Theta_5=0$

$$\delta_{V5} = \frac{M_p L^2}{24EI}$$

Correct Answer $= \dfrac{M_p L^2}{12EI}$ (Last hinge at Section 1)

3. CONTINUOUS BEAM DEFLECTION AT WORKING LOAD (Fig 30-a)

$$P_w = \frac{P_u}{F} = \frac{16 M_p}{L}\left(\frac{1}{1.75}\right)$$

$$P_w = 9.14\frac{M_p}{L} \qquad \left(\text{Note } P_y = 12\frac{M_p}{L}\right)$$

$$\delta_w = \frac{P_w L^3}{384EI}$$

$$\boxed{\delta_w = .024\frac{M_p L^2}{EI}}$$

4. LOAD-DEFLECTION CURVE FOR FIXED-ENDED BEAM (Fig 32)

[illegible]

In the elastic range,

[illegible]

[illegible] the shape of the [illegible] curve [illegible] simple beam, uniformly loaded, [illegible]

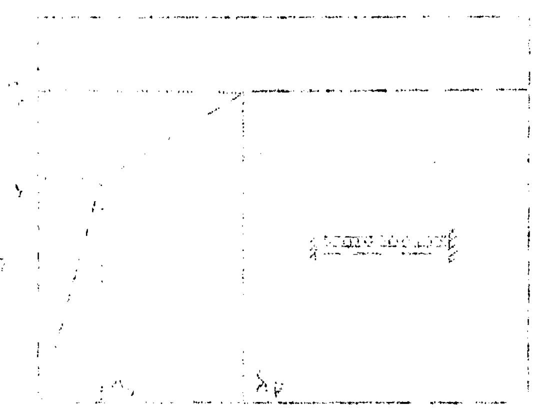


Fig.32

[illegible]

The slope of the [illegible] portion is [illegible] of the elastic part.

[illegible] $= \dfrac{M_p L^2}{32EI}$

[illegible]

$$\boxed{\delta_p = \frac{M_p L^2}{12EI}}$$

(Note check with example [illegible]

THE MOMENT - ϕ CURVE FOR WF SHAPES.

Average curve plotted for beams (solid) and columns (dotted). Variation in both are shown. Lightest and heaviest in each series are included. No Jr. beams or I's included. Columns b/d = $\frac{1}{2}$

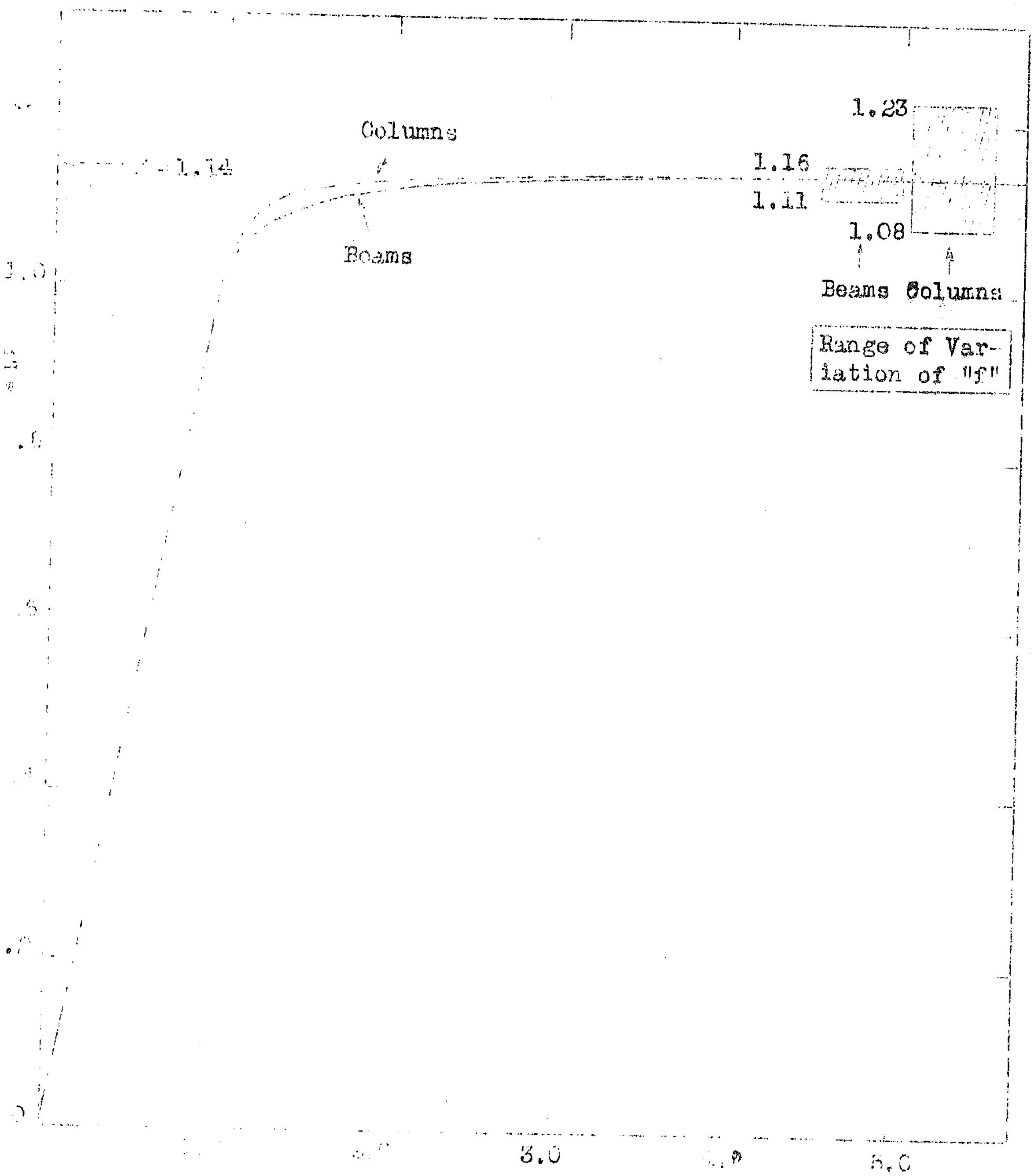


Fig. 35